Name:

Final Exam - PHY 4604 - Fall 2002
Wednesday, December 18, 2002

This exam is closed book and notes. You are not allowed (nor will you need) a calculator. Please use the space provided on the exam to do the problems. You may also use the backs of pages if additional space is needed.

1. **Short answer section**

   (a) What is Planck's constant numerically? Make sure to specify your units.

   (b) What is the time dependent Schrodinger equation?

   (c) What is the definition of the Hermitian conjugate of an operator A?

   (d) Express the time dependent Heisenberg operator, $A(t)$, in terms of the time independent Schrodinger operator, A.

   (e) What are the bound state energies of the hydrogen atom?

2. **One Dimensional Potentials**

Consider the one dimensional potential $V(x) = V_0$ for $x < 0$ and $V(x) = V_1$ for $x > 0$. Suppose that the energy, E, is greater than both V_0 and V_1.

(a) What is the form of the solution to the time independent Schrodinger equation for $x < 0$?

(b) What is the form of the solution to the time independent Schrodinger equation for $x > 0$?

(c) What are the boundary conditions at $x = 0$?

(d) Solve for the wave function for a wave coming from the left.

(e) Determine the transmission and reflection probabilities. Is probability conserved?

3. **Simple Harmonic Oscillator & Time Dependence**

Consider the harmonic oscillator hamiltonian,

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2.$$

The state of the system at $t = 0$ is

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\phi}}{\sqrt{2}}|1\rangle.$$

(a) What is the wave function at time t: $|\psi(t)\rangle$?

(b) What is the expectation of the position operator at time t, $\langle x\rangle_t$? The position operator for the harmonic oscillator is $x = \sqrt{\hbar/2m\omega}(a + a^\dagger)$.

(c) What is the expectation of the momentum at time t, $\langle p \rangle_t$? The momentum operator for the harmonic oscillator is $p = \sqrt{m\hbar\omega/2}i(a^\dagger - a)$.

(d) Test the uncertainty principle for $|\psi(t)\rangle$.

4. **Spin**

Consider a spin 1/2 system. For your reference the Pauli matrices are

$$\begin{aligned}
\sigma_x &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\
\sigma_y &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\
\sigma_z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\end{aligned}$$

(a) What are the eigenvalues and eigenvectors of the operator $(S_x + S_y)/\sqrt{2}$?

(b) Suppose a measurement of this operator is made, and the system is found to be in the state corresponding to the larger eigenvalue. What is the probability that a subsequent measurement of S_z yields $\hbar/2$?

(c) How would the result of part (b) change if we measure S_x instead of S_z?

5. **Orthogonality & Completeness**

Consider a particle in a box between $x = a$ and $x = 2a$. In other words the potential is zero for $a < x < 2a$, and infinite elsewhere.

(a) Sketch the wave functions for the lowest three eigenstates.

(b) What are the eigenstates and eigenvalues of the hamiltonian for this problem? Make sure to include the correct normalization constant.

(c) Show that the ground state and the first excited states are orthogonal.

(d) Assuming that the eigenstates of the hamiltonian form a complete basis, express the function $f(x) = 1$ for $a < x < 1.5a$ and zero otherwise as a linear combination of the eigenstates. Make sure to determine the coefficients used in the linear combination.

6. **Angular Momentum**

(a) For $l = 1$ determine the matrix representation of the operator L_y.

(b) What is the eigenvector of L_y with zero eigenvalue?

(c) Assuming one is in the zero eigenvalue state of part (b), what is the probability that a measurement of L_z will also yield zero?

(d) What is the probability that a measurement of L_z will yield $\hbar$ instead of 0?