Homework Assignment 10 - PHY 4604 - Fall 2002
(due Monday, Dec. 2 at the beginning of class)

1. **Spherical Bessel Functions:**

   (a) Consider the following potential in three dimensions: $V(r) = \infty$ for $r < a$ and $V(r) = 0$ for $r > a$. Note this is **not** the case of a particle in a well discussed in class, which has $V(r) = \infty$ for $r > a$ and $V(r) = 0$ for $r < a$. Using the spherical Bessel functions what is the general form for the radial wave function, $R(r)$, for angular momentum l and energy E?

   (b) What is the boundary condition at $r = a$? Apply this boundary condition to eliminate one of the constants in the wavefunction from part (a). (The other constant may be eliminated by the normalization condition.)

   (c) What is the asymptotic form of $R(r)$ for large r?

   (d) Now consider the potential: $V(r) = \infty$ for $r < a$, $V(r) = 0$ for $a < r < b$, and $V(r) = \infty$ for $r > b$. Again using the spherical Bessel functions, what is the general form for the radial wave function, $R(r)$, for angular momentum l and energy E?

   (e) What are the boundary conditions at $r = a$ and $r = b$?

   (f) For angular momentum zero, $l = 0$, write down the boundary conditions explicitly and solve for allowed energies. (This can be done analytically.)

2. **Hydrogen atom:**

   Solve for the radial wave function of the hydrogen atom for $n = 2$. There are two possible l values for $n = 2$: $l = 0$ and $l = 1$.

   (a) The first step is to solve for the function $y(\rho)$,

   $$y(\rho) = \rho^{l+1} \sum_{q=0}^{\infty} c_q \rho^q,$$

   where $\rho = r/a_o$ and the c_q are determined by the recursion relation

   $$c_q = \frac{2(\lambda(q+l) - 1)}{q(q+2l+1)} c_{q-1}$$

   with $\lambda = 1/n$. For $n = 2$ and $l = 1$ show that $c_1 = 0$. For $n = 2$ and $l = 0$ show that $c_2 = 0$, and determine the ratio of c_1/c_0. This will allow you to express $y(\rho)$ in terms of the c_o in both cases.

   (b) From $y(\rho)$ determine the function $u(r)$ by

   $$u(r) = y(r/a_o) e^{-\lambda r/a_o}.$$

   (c) Determine c_o by the normalization condition

   $$\int_0^{\infty} (u(r))^2 = 1.$$

   (d) Finally compute $R(r) = u(r)/r$ and compare your results to those in the book on page 208.