Homework 1:

1. Planck's formula for the energy density of a black body is

$$u(\nu, T) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/kT} - 1},$$

where
$$h = 6.63\times10^{-27}\,\text{erg sec} = 6.63\times10^{-34}\,\text{J}\cdot\text{s}$$
$$k = 1.38\times10^{-16}\,\text{erg/deg} = 1.38\times10^{-23}\,\text{J's}$$
$$c = 3.00\times10^{10}\,\text{cm/sec} = 3.00\times10^{8}\,\text{m/s}$$

$$\rightarrow \frac{8\pi h}{c^3} = \frac{6.63\times10^{-27}\times 8\pi}{(3\times10^{10})^3}\,\frac{\text{erg}\cdot\text{s}^4}{\text{cm}^3}$$
$$= 6.17\times10^{-57}\,\frac{\text{erg}\cdot\text{s}^4}{\text{cm}^3} = 6.17\times10^{-56}\,\frac{\text{J}\cdot\text{s}^4}{\text{m}^3}$$

$$\frac{k\cdot 300\text{K}}{h} = \frac{(1.38\times10^{-16})(300\text{K})}{6.63\times10^{-27}}\,\frac{\text{erg}}{\text{erg}\cdot\text{sec}} = 6.24\times10^{12}\,\text{Hz}$$

$$\frac{k\cdot 3\text{K}}{h} = 6.24\times10^{10}\,\text{Hz}$$

The plots are shown on the following pages, and a matlab file which produces these plots is given.

For $T = 300\text{K}$, the frequency at the maximum is $\approx 1.7\times10^{13}$ Hz while for $T = 3\text{K}$ it is at 1.7×10^{11} Hz. These wavelengths are approximately 1.8×10^{-5} m = 18000 nm (infrared) and 1.8 mm (microwave).

2. The DeBroglie wavelength is $\lambda = h/p \quad (p = h/\lambda)$.

(a) $1\,eV = 1.6 \times 10^{-12} erg = \dfrac{p^2}{2(0.9 \times 10^{-27} g)}$

$\rightarrow p = 5.367 \times 10^{-20}\, g\,\frac{cm}{s} = 6.63 \times 10^{-27}\, g\,\frac{cm^2}{s}\,\frac{1}{\lambda}$

$\rightarrow \lambda = 1.2 \times 10^{-7}\,cm = 12\,A^\circ$

(b) $10\,MeV = 1.6 \times 10^{-5}\,erg = \dfrac{p^2}{2(1.67 \times 10^{-24} g)}$

$\rightarrow p = 7.31 \times 10^{-15}\, g\,\frac{cm}{s} = 6.63 \times 10^{-27}\, g\,\frac{cm^2}{s}\,\frac{1}{\lambda}$

$\rightarrow \lambda = 9.1 \times 10^{-13}\,cm$

(c) $100\,MeV = 1.6 \times 10^{-4} erg = [(m_o c^2)^2 + (pc)^2]^{1/2}$

$m_o c^2 = (0.9 \times 10^{-27} g)(3 \times 10^{10}\,\frac{cm}{s})^2 = 8.1 \times 10^{-7} erg$

$\rightarrow (pc)^2 = (1.6 \times 10^{-4} erg)^2 - (8.1 \times 10^{-7} erg)^2$

$\rightarrow pc \approx 1.6 \times 10^{-4} erg \rightarrow p = 5.33 \times 10^{-15}\,\frac{g \cdot cm}{s}$

$\rightarrow \lambda = 1.24 \times 10^{-12}\,cm = 1.24 \times 10^{-14}\,m$

(d) $\dfrac{3 \cdot k\,T}{2} = \dfrac{3 \cdot 1.38 \times 10^{-16} \cdot 300}{2}\,erg = 6.2 \times 10^{-14} erg = \dfrac{p^2}{2m}$

$\rightarrow p = 4.55 \times 10^{-19}\, g\,\frac{cm}{s} = 6.63 \times 10^{-27}\, g\,\frac{cm^2}{s}\,\frac{1}{\lambda}$

$\rightarrow \lambda = 1.5 \times 10^{-8}\,cm = 1.5\,A^\circ$

3. $E = \frac{p^2}{2m} = \frac{h^2}{2m}\frac{1}{\lambda^2} = \frac{(6.63\times10^{-27}\,erg\,sec)^2}{2(0.9\times10^{-27}\,g)}\frac{1}{\lambda^2}$

$= 2.442\times10^{-26}\,erg\,(\frac{cm}{\lambda})^2$

(a) $\lambda = 150 A^\circ \Rightarrow E = 2.442\times10^{-26}\,erg\,\frac{(cm)^2}{(150\times10^{-8}\,cm)^2}$

$= 1.085\times10^{-14}\,erg$

$= 6.8\,meV$

(b) $\lambda = 5 A^\circ \Rightarrow E = 9.768\times10^{-12}\,erg$

$= 6.1\,eV$

4. $\left.\begin{array}{l} 2\pi r = n\lambda \text{ for } n = 1, 2, 3, \ldots \\ p = \frac{h}{\lambda} = mv \rightarrow \lambda = \frac{h}{mv} \end{array}\right\} 2\pi r = n\frac{h}{mv}$

$\rightarrow mvr = n\hbar$

5. $\lambda = 0.5 A^\circ$. In Bragg reflection constructive interference occurs at

$n\lambda = 2a\sin\theta \rightarrow \sin\theta = \frac{n\lambda}{2a}$

The ratio of $\sin\theta$ for the first & second maxima is

$\frac{\sin\theta_2}{\sin\theta_1} = 2 \rightarrow \theta_2 = \sin^{-1}(2\cdot\sin 5^\circ) \approx 10^\circ.$

6. $mvr = n\hbar$ and $\frac{mv^2}{r} = m\omega^2 r$

$\hookrightarrow \frac{v}{r} = \omega$

$\hookrightarrow mr^2\omega = n\hbar$

$\hookrightarrow r = \sqrt{\frac{n\hbar}{m\omega}}$ and $v = \sqrt{\frac{n\hbar\omega}{m}}$

$\Rightarrow E = \frac{1}{2}mv^2 + \frac{m\omega^2 r^2}{2} = \frac{1}{2}n\hbar\omega + \frac{1}{2}n\hbar\omega = \boxed{n\hbar\omega = E_n}$

The classical radiation frequency is $\frac{v}{2\pi r} = \frac{\omega}{2\pi}$.

The quantum radiation frequency is

$E_{n+1} - E_n = (n+1)\hbar\omega - n\hbar\omega = \hbar\omega,$

which is exactly the classical value for all n.

7. (a) $z_1 + z_2 = 4 - 2i$

(b) $z_1 - z_2 = -2 + 6i$

(c) $z_1 * z_2 = (1+2i)(3-4i) = (3+8) + i(6-4) = 11 + 2i$

(d) $\frac{z_1}{z_2} = \frac{1+2i}{3-4i}\,\frac{3+4i}{3+4i} = \frac{(3-8)+i(6+4)}{9+16} = \frac{-5+10i}{25}$

$= \frac{-1}{5} + i\frac{2}{5}$

(e) $z_1^* = 1 - 2i$

(f) $|z_1|^2 = 1 + 4 = 5$

(g) $|z_1| = \sqrt{5}$

(h) $\exp(z_1) = \exp(1+2i) = e^1 e^{2i} = e^1(\cos 2 + i\sin 2)$

$= 2.718(-0.416 + i\,0.909) \approx -1.13 + i\,2.47$

(i) $\sin(z_1) = \sin(1+2i) = \sin(1)\cos(2i) + \cos(1)\sin(2i)$

$= \sin(1)\cosh(2) + \cos(1)\frac{1}{i}(-\sinh(2))$

$= \sin(1)\cosh(2) + i\cos(1)\sinh(2)$

$\approx 3.17 + i$

8. $$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$e^{ix} = 1 + ix - \frac{x^2}{2!} - i\frac{x^3}{3!} + \frac{x^4}{4!} + i\frac{x^5}{5!} + \cdots$$

$$= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)$$

$$= \cos x + i\sin x \quad //$$

$$e^{ix} + e^{-ix} = (\cos x + i\sin x) + (\cos x - i\sin x) = 2\cos x$$

$$e^{ix} - e^{-ix} = (\quad " \quad\quad " \quad) - (\quad " \quad\quad " \quad) = 2i\sin x$$

$$\longrightarrow \cos x = \frac{e^{ix} + e^{-ix}}{2} \text{ and } \sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad //$$

hw1.m

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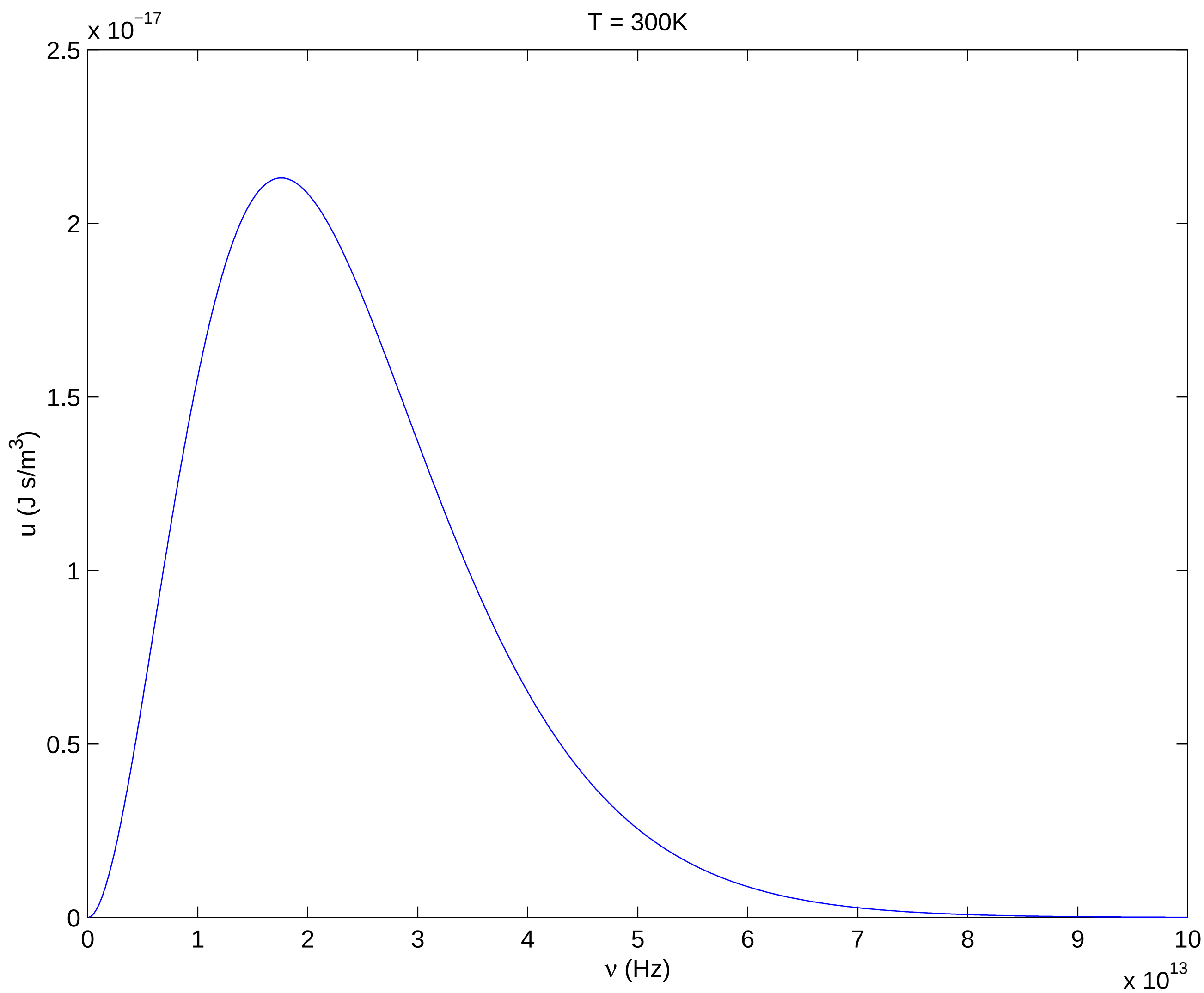
%matlab file for Homework 1

prefactor = 6.17e-56;
nu1 = 6.24e12;
nu2 = 6.24e10;

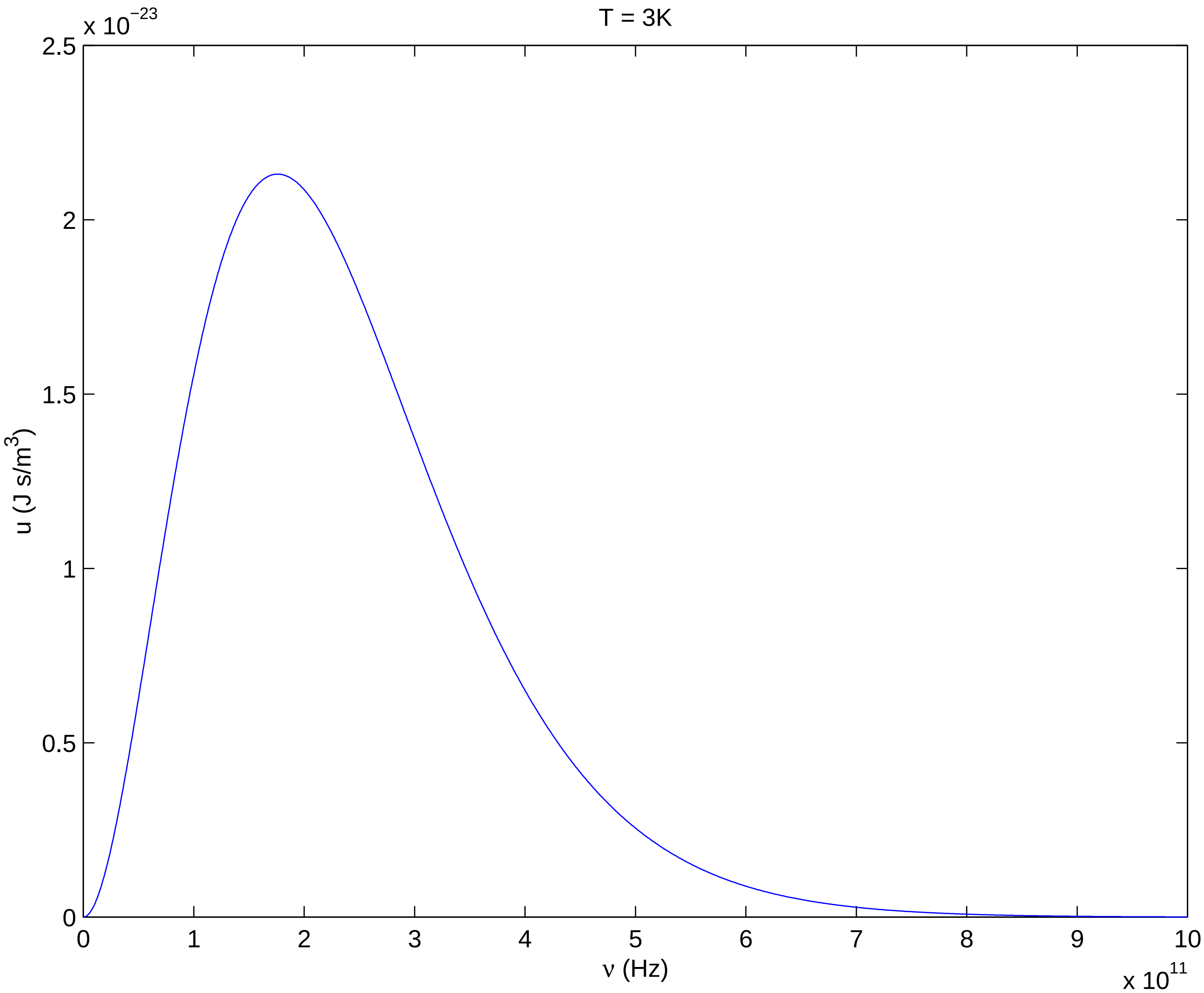
freq1 = 1e11:1e11:1e14;
u1 = prefactor*(freq1.^3)./(exp(freq1/nu1)-1);
figure(1)
plot(freq1,u1)
xlabel('\nu (Hz)')
ylabel('u (J s/m^3)')
title('T = 300K')

freq2 = 1e9:1e9:1e12;
u2 = prefactor*(freq2.^3)./(exp(freq2/nu2)-1);
figure(2)
plot(freq2,u2)
xlabel('\nu (Hz)')
ylabel('u (J s/m^3)')
title('T = 3K')

[Y,I] = max(u1); freq1(I)
[Y,I] = max(u2); freq2(I)
```



T = 300K
x 10^-17
2.5
2
1.5
1
0.5
0
u (J s/m^3)
0
1
2
3
4
5
6
7
8
9
10
ν (Hz)
x 10^13


T = 3K
x 10^-23
u (J s/m^3)
2.5
2
1.5
1
0.5
0
0 1 2 3 4 5 6 7 8 9 10
ν (Hz)
x 10^11