Homework Assignment 2 - PHY 4604 - Fall 2002
(due Friday September 13 at the **beginning** of class)

In this homework assignment we (1) study probability conservation, (2-3) practice computing expectation values and Fourier transforms, (4) prove x and p are Hermitian operators, and (5) plot an example of a delta function. Do not wait until the last minute to work on this assignment.

1. Suppose that the potential $V(x)$ is complex. Obtain an expression for $\partial P(x,t)/\partial t$ and $d/dt \int dx P(x,t)$. For absorption, the last must be negative. What does this tell us about $V(x)$?

2. The integral of a Gaussian function, $\exp(-\alpha x^2)$, is

$$\int_{-\infty}^{\infty} dx e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}. \tag{1}$$

   (a) Suppose that $\psi(x) = Ce^{-\alpha x^2/2}$. Determine C so that this wave function is normalized.
   (b) Compute the expectation value, $\langle x^n \rangle$. Hint: For odd n you can get this by symmetry, and for even n consider taking a partial derivative with respect to α.
   (c) Compute $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$.
   (d) Compute the Fourier transform $\psi(p)$. Hint: Complete the square in the exponential.
   (e) Using $\psi(p)$ determine $\langle p^n \rangle$.
   (f) Compute $\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$.
   (g) Calculate $\Delta x \Delta p$ and compare it to the uncertainty relation.

3. Consider the wavefunction $\psi(x) = Ce^{-\lambda|x|}$.

   (a) Determine C necessary to normalize the wave function.
   (b) Compute the uncertainty in x: $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$.
   (c) Compute the Fourier transform $\psi(p)$.
   (d) From the wave function in momentum space compute the uncertainty in the momentum: $\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$.
   (e) Compare $\Delta x \Delta p$ to the uncertainty relation.

4. An operator whose expectation value is real for all wave functions is called a hermitian operator.

   (a) Show that the position operator, x, is hermitian by showing that $\langle x \rangle = \langle x \rangle^*$.
   (b) Show that the momentum operator, p, is hermitian by showing that $\langle p \rangle = \langle p \rangle^*$.
   (c) Consider the functions $\psi(\theta)$ of the angular variable θ, restricted to the interval $-\pi \leq \theta \leq \pi$ and satisfying $\psi(\pi) = \psi(-\pi)$. Show that the operator

$$L = \frac{\hbar}{i}\frac{d}{d\theta} \tag{2}$$

   is hermitian.

5. Plot the function

$$f(x) = \frac{\sin(Nx)}{\pi x} \tag{3}$$

   for $N = 1, 10, 100$. Show that the integral $\int dx f(x)$ is independent of N. Evaluate $\int dx f(x)$ numerically for $N = 1$.