Homework Assignment 3 - PHY 4604 - Fall 2002
(due Friday September 20 at the **beginning** of class)

In the previous homework we practiced taking expectation values of different wave functions. The focus of this homework assignment is on what the book calls the expansion hypothesis. The integrals are are not as complicated last week, but there is more numerical work.

1. Which of the following operators is linear?

$$\begin{aligned}
\mathcal{O}_1\psi(x) &= x^3\psi(x) \\
\mathcal{O}_2\psi(x) &= x\frac{d}{dx}\psi(x) \\
\mathcal{O}_3\psi(x) &= \lambda\psi(x)^* \\
\mathcal{O}_4\psi(x) &= e^{\psi(x)} \\
\mathcal{O}_5\psi(x) &= \frac{d}{dx}\psi(x) + a \\
\mathcal{O}_6\psi(x) &= \int_{-\infty}^{x} dx'(\psi(x')x')
\end{aligned}$$

2. Calculate the following commutators.

   (a) $[\mathcal{O}_2, \mathcal{O}_6]$

   (b) $[\mathcal{O}_1, \mathcal{O}_2]$

3. In class and in the book the eigenfunctions of the particle in a box problem are shown to be the functions $u_n^{(\pm)}(x)$. According to the expansion postulate (or completeness) any function, $f(x)$, can be written in the form

$$f(x) = \sum_{n=1}^{\infty}[A_n^{(+)}u_n^{(+)}(x) + A_n^{(-)}u_n^{(-)}(x)]. \tag{1}$$

   Determine the $A_n^{(+)}$ and $A_n^{(-)}$ for the functions

   (a) $f(x) = 1$ for $-a/2 < x < a/2$ and $f(x) = 0$ elsewhere,

   (b) $f(x) = \sin(3\pi x/a)$,

   (c) $f(x) = a^2 - x^2$.

4. Using the results from the previous question, plot the sum

$$\sum_{n=1}^{5}[A_n^{(+)}u_n^{(+)}(x) + A_n^{(-)}u_n^{(-)}(x)] \tag{2}$$

   and compare it to the exact answer for each of the cases (a)-(c). Set $a = 1$ in plotting the functions.