Homework Assignment 6 - PHY 4604 - Fall 2002
(due Friday October 18 at the **beginning** of class)

1. The eigenstates of a particle in a infinite box between 0 and a are given by

$$\langle x|n\rangle = u_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi n x}{a}\right). \tag{1}$$

   (a) Given that the hamiltonian inside the box is $H = -(\hbar^2/2m)d^2/dx^2$ determine the eigenvalues, E_n, of H for the states $u_n(x)$.

   (b) Suppose that at $t = 0$ the state of the system is

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}|1\rangle + \frac{i}{\sqrt{2}}|2\rangle. \tag{2}$$

   What is $\psi(x,t) = \langle x|\psi(t)\rangle$ at an arbitrary time t?

   (c) This wave function, $\psi(x,t)$ is periodic in time. What is the period, T?

   (d) Plot the probability density, $|\psi(x,t)|^2$, for $t = 0$, $T/4$, $T/2$, and $3T/4$. Can you see how the particle is bouncing back and forth in the box?

   (e) Compute the matrix elements $\langle m|x|n\rangle$ and $\langle n|p|n\rangle$ for all four combinations of $m, n = 1, 2$.

   (f) Use the results for these matrix elements to compute

$$\langle x\rangle_t = \langle\psi(t)|x|\psi(t)\rangle \tag{3}$$
$$\langle p\rangle_t = \langle\psi(t)|p|\psi(t)\rangle. \tag{4}$$

   (g) Does the result for $\langle x\rangle_t$ agree with the differential equation for $d\langle x\rangle/dt$?

2. Define an operator B as $Bf(x) = f(a-x)$.

   (a) Show that B commutes with the Hamiltonian for a particle in a box between 0 and a.

   (b) Is B hermitian?

   (c) Show that $B^2 = 1$.

   (d) What are the possible eigenvalues of B?

   (e) Given the results of (a) - (d), what are the possibilities for $B|n\rangle$? Test your conclusions explicitly for $n = 1, 2$.

3. The hamiltonian for the harmonic oscillator is

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2. \tag{5}$$

   (a) Derive the equations of motion for the Heisenberg operators $x(t)$ and $p(t)$.

   (b) Solve these equations of motion for arbitrary initial operators $x(0)$ and $p(0)$.

   (c) Based on the results in part (b), give expressions for $\langle x\rangle_t$ and $\langle p\rangle_t$ in terms of $\langle x\rangle_{t=0}$ and $\langle p\rangle_{t=0}$.