Homework Assignment 9 - PHY 4604 - Fall 2002
(due Wednesday, Nov. 20 at the beginning of class)

1. Spherical harmonics:

   (a) Using the lowering operator

   $$L_- = \hbar e^{-i\phi}\left(-\frac{\partial}{\partial\theta} + i\cot(\theta)\frac{\partial}{\partial\phi}\right)$$

   and the spherical harmonic for $m = l$

   $$Y_l^l(\theta,\phi) = \frac{(-1)^l}{2^l l!}\sqrt{\frac{(2l+1)!}{4\pi}}(\sin\theta)^l e^{il\phi}$$

   determine all the spherical harmonics, $Y_l^m(\theta,\phi)$, for $l = 0$, 1, and 2. You can check your results with those on page 197.

   (b) Verify the normalization of the $l = 2$ spherical harmonics:

   $$\langle l=2,m|l=2,m\rangle = \int_0^\pi \sin(\theta)d\theta \int_0^{2\pi} d\phi |Y_2^m(\theta,\phi)|^2 = 1.$$

   Hint: do a change of variables to $u = \cos(\theta)$.

   (c) Verify the orthogonality of the $l = 1$ spherical harmonics:

   $$\langle l=1,m'|l=1,m\rangle = \int_0^\pi \sin(\theta)d\theta \int_0^{2\pi} d\phi Y_1^{m'*}(\theta,\phi)Y_1^m(\theta,\phi) = 0.$$

   for $m \neq m'$.

2. Problem 2, Chapter 11

3. Problem 3, Chapter 11

4. Problem 4, Chapter 11