

# Comparison Particle in a Box and Free Particle Cases

## Particle in a Box [0, a]

Eigenfunctions  $\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$

Orthonormal  $\int_0^a dx \psi_n(x) \psi_m(x) = \delta_{nm}$

Completeness  $\sum_{n=1}^{\infty} \psi_n^*(x) \psi_n(x') = \delta(x-x')$

"Fourier Analysis"  
 $f(x) = \sum_{n=1}^{\infty} A_n \psi_n(x)$   
 $A_n = \int_0^a dx \psi_n(x) f(x)$

## Free Particle ( $k = p/\hbar$ )

$$\psi_k(x) = e^{ikx}$$

$$\int_{-\infty}^{+\infty} dx \psi_k^*(x) \psi_{k'}(x) = 2\pi \delta(k-k')$$

$$\int_{-\infty}^{+\infty} \frac{dk}{2\pi} \psi_k^*(x) \psi_k(x') = \delta(x-x')$$

$$f(x) = \int_{-\infty}^{+\infty} \frac{dk}{2\pi} A(k) \psi_k(x)$$
$$A(k) = \int_{-\infty}^{+\infty} dx \psi_k^*(x) f(x)$$