Delta function potential:

$$V(x) = \underbrace{-\frac{\hbar^2}{2ma^2}}_{\text{energy}} \underset{\text{dim.less}}{\lambda} \underbrace{a\,\delta(x)}_{\text{dim.less}} = \frac{-\hbar^2}{2ma}\lambda\,\delta(x)$$

$$E\,u(x) = \frac{-\hbar^2}{2m}\frac{d^2u}{dx^2} - \frac{\hbar^2}{2ma}\lambda\delta(x)$$

$$\rightarrow \frac{d^2u}{dx^2} + \frac{2mE}{\hbar^2}u = -\frac{\lambda}{a}\delta(x)u$$

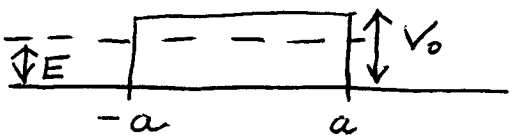
$$\int_{0^-}^{0^+} dx\,\frac{d^2u}{dx^2} + \int_{0^-}^{0^+} dx\,\frac{2mE}{\hbar^2}u = -\frac{\lambda}{a}\int_{0^-}^{0^+} dx\,\delta(x)\,u$$

$$\rightarrow \boxed{\frac{du}{dx}(0^+) - \frac{du}{dx}(0^-) = -\frac{\lambda}{a}u(0)}$$

The wave function is continuous, but the derivative is not.

Tunneling Phenomena:

Last time:



$$|T|^2 = \frac{(2kK)^2}{(k^2+K^2)^2 \sinh^2 2Ka + (2kK)^2}$$

For $2Ka \gg 1$,

$$|T|^2 \approx \left(\frac{4kK}{k^2+K^2}\right)^2 e^{-4Ka}, \text{ where } Ka = \left[\frac{2ma^2}{\hbar^2}(V_0 - E)\right]^{1/2}$$

$$2Ka = 2a\left[\frac{2m}{\hbar^2}(V_0 - E)\right]^{1/2}$$

$$\rightarrow |T|^2 \sim e^{-2\cdot 2a\sqrt{\frac{2m}{\hbar^2}(V_0 - E)}}$$

$$\sim e^{-2\int dx \sqrt{\frac{2m}{\hbar}(V(x) - E)}}$$ ... WKB approximation (next semester)

Application: STM (Scanning Tunneling Microscope)

<u>See</u>: www.nobel.se/physics/educational/microscopes/scanning/index.html