

3D vectorsWavefunction Vectors (discrete)Wavefunction (continuum)  
example

- Fourier Analysis

Addition/  
Subtraction

$$\vec{A} + \vec{B}$$

$$\psi_a(x) + \psi_b(x)$$

Scalar Mult.

$$c\vec{A}$$

$$c\psi_a(x)$$

Dot Product

$$\vec{A} \cdot \vec{B} \geq 0$$

$$\int dx \psi_a^*(x) \psi_b(x)$$

$$\vec{A} \cdot \vec{A} \geq 0$$

$$\int dx \psi_a^*(x) \psi_a(x) \geq 0$$

Basis

$$\hat{x}, \hat{y}, \hat{z}$$

$$u_n(x)$$

$$u_k(x) = e^{ikx}$$

Orthonormal

$$\hat{x} \cdot \hat{x} = 1 \dots$$

$$\int dx u_n(x)^* u_m(x) = \delta_{nm}$$

$$\hat{x} \cdot \hat{y} = 0 \dots$$

$$\int dx u_k(x)^* u_{k'}(x) = 2\pi \delta(k-k')$$

Components

$$A_x = \hat{x} \cdot \vec{A}$$

$$A_n = \int dx' u_n(x')^* \psi_a(x')$$

$$A_k = \int dx' u_k(x')^* \psi(x')$$

Basis  
Notation

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

$$\psi_a(x) = \sum_{n=1}^{\infty} A_n u_n(x)$$

$$\psi(x) = \int \frac{dk}{2\pi} A_k u_k(x)$$

$$= \hat{x} \hat{x} \cdot \vec{A} + \hat{y} \hat{y} \cdot \vec{A} + \dots$$

$$= \int dx' \left( \sum_{n=1}^{\infty} u_n(x')^* u_n(x) \right) \psi_a(x')$$

Completeness

$$\mathbb{1} = \hat{x} \hat{x} \cdot + \hat{y} \hat{y} \cdot + \hat{z} \hat{z} \cdot$$

$$\delta(x-x') = \sum_{n=1}^{\infty} u_n(x')^* u_n(x)$$

$$\delta(x-x') = \int \frac{dk}{2\pi} u_k(x')^* u_k(x)$$

## Dirac Notation

$$|a\rangle + |b\rangle$$

$$c|a\rangle$$

$$\langle a|b\rangle$$

$$|n\rangle$$

$$\langle n|m\rangle = \delta_{nm}$$

$$A_n = \langle n|a\rangle$$

$$|a\rangle = \sum_{n=1}^{\infty} |n\rangle \langle n|a\rangle$$

$$\mathbf{1} = \sum_{n=1}^{\infty} |n\rangle \langle n|$$

## Real Space Example

$$|x_0\rangle = \delta(x - x_0)$$

$$\langle x_0|m\rangle = \int dx \delta(x - x_0)^* u_m(x) = u_m(x_0)$$

$$\psi_a(x) = \langle x|a\rangle$$

$$\begin{aligned} |a\rangle &= \int dx |x\rangle \langle x|a\rangle \\ &= \int dx \psi_a(x) |x\rangle \end{aligned}$$

$$\mathbf{1} = \int dx |x\rangle \langle x|$$

$$\delta(x - x') = \langle x|x'\rangle = \int dx'' \delta(x - x'') \delta(x' - x'')$$