Study Guide/Practice Exam - PHY 4604 - Fall 2002

This study guide/practice exam contains some sample exam questions. As a motivation, if you complete it and hand it in to me in class on Friday, I will give you 3 bonus points on the exam. There will be three parts to the exam: (1) a short answer section, (2) a problem on computing expectation values or (3) the expansion hypothesis, and (4) a problem on solving the Schrodinger equation for piecewise constant potentials in one dimension. You are **not** allowed any formula sheets or a calculator on the exam. **Remember the exam date has been moved from Friday to Monday.**

1. Short answer section. I will choose several of these kinds of questions. Here is a sampling of possible questions.

   (a) Name an experiment which indicates that matter has wave properties.

   (b) Name an experiment which indicates that light has particle properties.

   (c) What is Bohr's quantization condition?

   (d) What is the correspondence principle?

   (e) What is Planck's constant numerically?

   (f) What is the time-dependent Schrodinger equation?

   (g) What is the time-independent Schrodinger equation?

   (h) What is the continuity equation?

   (i) What is the probability current in one dimension?

   (j) Let $\psi(x)$ be a wave function on the interval $-\infty < x < \infty$. Give expressions for the expectation value of x and p.

   (k) What is the Fourier transport of $\psi(x)$ in momentum space ($\psi(p)$)?

   (l) Give expressions for the expectation values of x and p using $\psi(p)$.

   (m) Given $\psi(p)$ how does one go back and determine $\psi(x)$.

   (n) What is the normalization condition for $\psi(x)$?

   (o) What is the normalization condition for $\psi(p)$?

   (p) What is the uncertainty principle?

   (q) What is the condition for a set of wave functions, $u_n(x)$, to be orthonormal?

   (r) A set of wave functions, $u_n(x)$, for $n = 1, 2, 3 \ldots$ on the interval $[a, b]$ is said to be complete or satisfy the expansion hypothesis. What does this mean?

   (s) In a region where the potential is constant, $V(x) = V_o$, what is the form of the solution to the time-independent Schrodinger equation if $E > V_o$?

   (t) What is the form if $E < V_o$?

   (u) For a piecewise constant potential in one dimension, what are the boundary conditions for the wave function at a step in the potential?

   (v) What are the boundary conditions for a wave function at a delta function potential?

2. The ground state of a particle in a box between $0 < x < a$ is $\psi(x) = C \sin(\pi x/a)$.

   (a) Determine the constant C from the normalization condition.

   (b) What is the expectation value of x? Hint: you can determine this by symmetry.

   (c) What is Δx? Here you will have to do an integral. On the exam if you can not do an integral, but set up the integral correctly, you will get partial credit.

   (d) What is the expectation value of the momentum?

   (e) What is Δp?

   (f) Compare your results with Δx and Δp to the uncertainty principle.

3. Consider the wave functions $u_n(x) = C_n \sin(n\pi x/a)$ on the interval $0 < x < a$.

   (a) Show that these wave functions are orthogonal.

   (b) Determine the normalization constants C_n.

   (c) These wave functions are complete on the interval $0 < x < a$. Let $f(x) = 1$ for $0 < x < 0.5a$ and 0 otherwise. Express $f(x)$ as a sum of the form

$$f(x) = \sum_{n=1}^{\infty} A_n u_n(x). \tag{1}$$

   (You need to determine the coefficients A_n).

4. Consider the potential in one dimension $V(x) = 0$ for $x < 0$, $V(x) = V_o$ for $x > 0$, and $V(x) = \lambda(\hbar^2/2ma)\delta(x)$ at $x = 0$. Suppose $E > V_o$.

   (a) What is the form of the solution to the time-independent Schrodinger equation for $x < 0$?

   (b) What is the form of the solution to the time-independent Schrodinger equation for $x > 0$?

   (c) What are the boundary conditions at $x = 0$?

   (d) Solve for the wave function for a wave coming **from the right**.

   (e) Determine the transmission and reflection probabilities. Is probability conserved?