Study Guide/Practice Exam 3

This exam will cover the Schrodinger equation in three dimensions, angular momentum, the hydrogen atom, matrix representations and spin. As before there are four parts: a short answer section, a problem on angular momentum, a problem on central potentials, and a problem on spin. With the exception of the short answer section, these are only representative problems. Please study the homework as well. Solutions to this practice exam and the homework will be available on-line shortly. Because we are short on time, you do not have to hand in this assignment.

Also, note that the final exam is on Wednesday, December 18 from 5:30 to 7:30 PM in our regular classroom.

1. **Short answer section**

   I will choose five of the following questions for the exam.

   (a) What are the commutators of L_x, L_y, L_z, and L^2?

   (b) Define L_+ and L_-. Express L_x and L_y in terms of L_+ and L_-.

   (c) What are the eigenvalues of the J^2 and J_z operators?

   (d) What is J_+ acting on $|j,m\rangle$? What is J_- acting on $|j,m\rangle$? What is J_z acting on $|j,m\rangle$?

   (e) What are the allowed values of L^2 and L_z for the spherical harmonics? Why?

   (f) What is the orthogonality condition for the spherical harmonics?

   (g) What is the completeness condition for the spherical harmonics?

   (h) What is the form of the solution to the Schrodinger equation for a central potential?

   (i) What is the radial Schrodinger equation for $R(r)$? What is the normalization condition for $R(r)$?

   (j) What is the radial Schrodinger equation for $u(r)$? How is $u(r)$ related to $R(r)$? What is the normalization condition for $u(r)$?

   (k) What is the behavior of $u(r)$ as $r \to 0$ for angular momentum l and a not too singular potential?

   (l) What are the solutions to the radial Schrodinger equation for a free particle ($V = 0$)? Give explicit expressions for these solutions for $l = 0$.

   (m) What is the Bohr radius (numerically)?

   (n) What is a Rydberg (numerically)?

   (o) What is the asymptotic behavior of the eigenstates of the hydrogen atom for energies less than zero?

   (p) What are the bound state energies of the hydrogen atom?

   (q) What are the degeneracies of the bound states of the hydrogen atom?

2. **Angular momentum**

   (a) Compute the commutator $[[L_+, L_z], L_-]$ and compare it to $[[L_+, L_-], L_z]$.

   (b) Expand the function, $f(\theta, \phi) = 1 + \sin(\theta)\sin(\phi + \frac{\pi}{4})$ in terms of the spherical harmonics. You may look in your book for the spherical harmonics. On the exam I would give you the spherical harmonics on a formula sheet. Do not leave your final answer in terms of integrals.

   (c) For $l = 1$ compute the matrix elements of the operator $L_z L_+$ and express it as a matrix.

3. **Central potentials**

   In this section you will solve a problem with a central potential. The following is one of the standard examples.

   A three dimensional harmonic oscillator has a Hamiltonian of

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 r^2.$$

   (a) What is the radial Schrodinger equation for $u(r)$ for this Hamiltonian and angular momentum l?

   (b) For large r what is the asymptotic form of the wave function? (Hint: At large r look for a solution of the radial Schrodinger equation of the form $\exp(f(r))$, where $f(r)$ is some function to be determined.)

   (c) What is the behavior of $u(r)$ as $r \to 0$ for a given l?

   (d) Show that the asymptotic solution is part (b) is an exact solution for $l = 0$. What is the energy of this state?

   (e) Show that the wave function $\psi(x, y, z) = X(x)Y(y)Z(z)$ is an eigenstate of this Hamiltonian when when $X(x)$, $Y(y)$, and $Z(z)$ are eigenstates of the *one* dimensional harmonic oscillator. From what you know about the harmonic oscillator, prove that the eigenstate in part (d) is the ground state, and it is non-degenerate.

4. **Spin**

   This problem will be similar to problems 7, 12, and 14 from Chapter 14 in the last homework assignment. I will post the solutions to these problems right after class on Friday so make sure to hand in your homework on time. I will also explain these problems in class on Friday.