ADMX Collaboration Meeting
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Low Frequency Project

Pierre Sikivie

• Motivation
• Reentrant Cavity simulations
• Prototype reentrant cavity

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caustic ring
Inner Galactic rotation curve

from Massachusetts-Stony Brook North Galactic Pane CO Survey (Clemens, 1985)
GAIA sky map
with respect to a non-rotating Galactic reference frame

<table>
<thead>
<tr>
<th>Flow</th>
<th>$d$ [$10^{-24}$ g/cm$^3$]</th>
<th>$v^G_\rho$ [km/s]</th>
<th>$v^G_\phi$ [km/s]</th>
<th>$v^G_z$ [km/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big</td>
<td>20.0</td>
<td>−104.4</td>
<td>509.4</td>
<td>6.1</td>
</tr>
<tr>
<td>Little</td>
<td>2.0</td>
<td>−0.2</td>
<td>520.0</td>
<td>4.5</td>
</tr>
<tr>
<td>Up</td>
<td>9.6</td>
<td>−115.3</td>
<td>505.1</td>
<td>44.8</td>
</tr>
<tr>
<td>Down</td>
<td>8.4</td>
<td>−116.4</td>
<td>505.4</td>
<td>−38.1</td>
</tr>
</tbody>
</table>

$\hat{z}$ is toward the North Galactic Pole

$\hat{\phi}$ is in the direction of Galactic rotation

$\hat{\rho}$ is in the direction away from the Galactic Center

Uncertainties on the densities are large but

$$d_{\text{Big}} > 10.0 \times 10^{-24} \frac{\text{gr}}{\text{cc}} = 6 \frac{\text{GeV}}{\text{cc}}$$
• The evidence for caustic rings requires that the dark matter has vorticity.

• Ordinary cold dark matter, such as WIMPs and sterile neutrinos, cannot have vorticity. Their velocity field is curl free.

• Axion dark matter acquires vorticity because it forms a rethermalizing Bose-Einstein condensate. Indeed the lowest energy state for given angular momentum is a state of rotation.
• The axion dark matter fluid first thermalizes (by gravitational self-interactions) and forms a BEC when the universe is approximately 1 year old (Q. Yang and PS, 2009; O. Erken et al., 2012)

• The horizon at age of one year expands to the size of the largest disk galaxies today.

• Assuming the above is not an accident, we have a new handle on the axion mass.
\[ m_a \sim \mu eV \left( \frac{0.27}{\Omega_a} \right)^{\frac{3}{2}} \]

242 MHz

\[
(2\pi) 120 \text{ MHz} \lesssim m_a \lesssim (2\pi) 3 \text{ GHz}
\]
A reentrant cavity search for axion dark matter

ADMX    Letter of Intent
          August 20, 2020

Abstract:

We propose to search for axion dark matter in the frequency range 100 - 625 MHz (0.4 < m_a < 2.6 μeV) using a reentrant cavity haloscope. First a small prototype will be built and tested in data-taking mode inside the magnet of the 1980’s UF Axion Project. Based on this experience, a larger reentrant cavity haloscope will be built for the ADMX site at UW and operated there.
Higher Order Reentrant Post Modes in Cylindrical Cavities

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(Dated: 4 December 2018)

Reentrant cavities are microwave resonant devices employed in a number of different areas of physics. They are appealing due to their simple frequency tuning mechanism, which offers large tuning ranges. Reentrant cavities are, in essence, 3D lumped LC circuits consisting of a conducting central post embedded in a resonant cavity. The lowest order reentrant mode (which transforms from the $TM_{010}$ mode) has been extensively studied in past publications. In this work we show the existence of higher order reentrant post modes (which transform from the $TM_{01n}$ mode family). We characterize these new modes in terms of their frequency tuning, filling factors and quality factors, as well as discuss some possible applications of these modes in fundamental physics tests. The appendix contains a comment on a paper related to this work.

1. INTRODUCTION

The cylindrical reentrant cavity is a device that can provide high-Q microwave modes with large tuning ranges. It consists of a metal cavity with a conducting post or ring located centrally within the cavity. The gap between the top of this post or ring and the top of the cavity adjusts the mode frequency and at certain gap spacings traps the electric field within the gap. Such cavity designs have been extensively studied and allow for a standard reentrant mode tuning range on the order of GHz without the need for physically large cavities.1-6

Since this structure was first investigated in connection with the development of klystrons, it has been widely used in the construction of microwave oscillators and particle accelerators7-8, and is often chosen as a structure in the study of metamaterials9-10. Some recent work has discussed potential applications in telecommunications11, and detection of gravitational waves12,13 and dark matter14. It is an interesting perspective to view the standard reentrant mode as a perturbed $TM_{010}$ mode. The standard reentrant mode transforms into the empty cavity $TM_{010}$ mode as the central post is removed from the cavity15. In a similar way, there exist higher order reentrant modes, which can be viewed as perturbed $TM_{01n}$ modes.

In this work we unequivocally demonstrate the existence of these higher order reentrant modes and characterize them in terms of their tuning range, and quality factor. Section II presents a theoretical study of these modes based on finite element analysis, and experimental data follows in Section III.

![Figure 1: Cylindrical cross-section of the cavity from the COMSOL model in the r-z plane (encompassing the origin) in cylindrical coordinates.](image)

1. Finite Element Analysis of the Reentrant Cavity

- Simulate the cavity using a finite element method (FEM) software like COMSOL or Ansys to model the electric field distribution and resonant frequencies.
- Compare the simulated results with theoretical predictions to validate the model.

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Reentrant Cavity
Possible Reentrant Cavity Design
Dielectrically loaded cavity simulations by Mohamed. See Ben’s talk.
Axially symmetric reentrant cavities simulations

\[ f/f_0 = 0.406, \alpha = 0.534 \]

\[ a(\rho, z) = \rho B(\rho, z) \]

\[ \vec{B} = B(\rho, z) \hat{\phi} \quad \vec{E} = \frac{i}{\omega} \left[ \left( \frac{\partial B}{\partial \rho} + \frac{B}{\rho} \right) \hat{z} - \frac{\partial B}{\partial z} \hat{\rho} \right] \]
Reentrant Cavity Form factor

\[
C(f) = C_0 \left( \frac{f}{f_0} \right)^2 \alpha \left( \frac{f}{f_0} \right)
\]

\[
C_0 = 0.692 \\
f_0 = \frac{2.4048}{2\pi R}
\]

\[
0.25 \leq \alpha \leq 0.95
\]

\[
f = 230 \text{ MHz} \\
f_0 = 570 \text{ MHz} \\
\alpha = 0.4 \\
C = 0.045
\]
For posts both of length 25 cm:

\[ \frac{f}{f_0} = 0.403, \alpha = 0.532 \]

For posts of lengths 25 cm on the left, 24 cm on the right:

\[ \frac{f}{f_0} = 0.405, \alpha = 0.304 \]
Sensitivity of an ADMX reentrant cavity search

\[ P = g^2 \rho B^2 V C \frac{1}{m} Q \]

\[ P = P_0 Q \]

\[ \frac{d \log(f)}{dt} = \frac{1}{f} \frac{1}{(s/n)^2} \left( \frac{P_0}{T_n} \right)^2 \frac{4}{9} Q a Q \]
\[ \delta v < 70 \text{ m/s} \simeq 2 \times 10^{-7} c \]

\[ \frac{\delta f}{f} \sim \frac{\nu \delta v}{c^2} < 2 \times 10^{-10} \]

For example

\[ f = 300 \text{ MHz} \]

\[ \delta f < 60 \text{ mHz} \]
No mode crossings.

With the ADMX magnet at CENPA, how low can we go?

\[ f_0 = 574 \text{ MHz} \]

\[ C(f) = 0.4 \left( \frac{f}{f_0} \right)^2 \]

\[ Q(f) \sim \frac{R}{\delta} \propto f^{\frac{1}{3}} \]
\[ T_n \sim f^0 \quad \text{(constant)} \]

sensitivity \( \propto \frac{g^2}{m_a} \rho \frac{\sqrt{Q_a}}{s/n} C \)

\[ 373 \left( \frac{f}{f_0} \right)^{2+1+\frac{1}{6}} = 1 \]

for Big Flow density of 6 GeV/cc

\[ f = 88.5 \text{ MHz} \]
Medres sensitivity for our standard 0.45 GeV/cc density.
Prototype reentrant cavity design by Joe Gleason
movable post insertion depths

- L: 100
  - 5, 10, 20, 30, 40, 50, 60, 65, 70

- 90
  - 5, 10, 20, 30, 40, 50, 55, 60

- 80
  - 5, 10, 20, 30, 40, 45, 50

Graph showing the ratio \( Q/Q_0 \) against \( f/f_0 \).
Conclusion:

a low frequency search is “low lying fruit”

Thank you