Announcements

Homework 2 due TODAY!

Homework 3 due on Feb 8 (next Wednesday).

Last time

- Photoelectric experiment
- What is an photon
- Kinetic energy of a photon
  $K_{max}=hf-\Phi$
- Stopping potential
  $K_{max}=eV_s$

Today

- Black body radiation

in-class quiz (3 min)

A light source shining on a metal surface causes photoelectrons to be emitted. If the source’s intensity is made 4x brighter, which statement is closest to being true?

a. Kmax unchanged, # emitted electrons is 2x

b. Kmax unchanged, # emitted electrons unchanged

c. Kmax is 4x, # emitted electrons unchanged

d. Kmax unchanged , # emitted electrons is 4x

e. Kmax is 2x , # emitted electrons unchanged

in-class quiz (3 min)

A light source shining on a metal surface causes photoelectrons to be emitted. If the source's intensity is made 4x brighter, which statement is closest to being true?

$$K_{max} = hf - \phi$$

a. Kmax unchanged, # emitted electrons is 2x

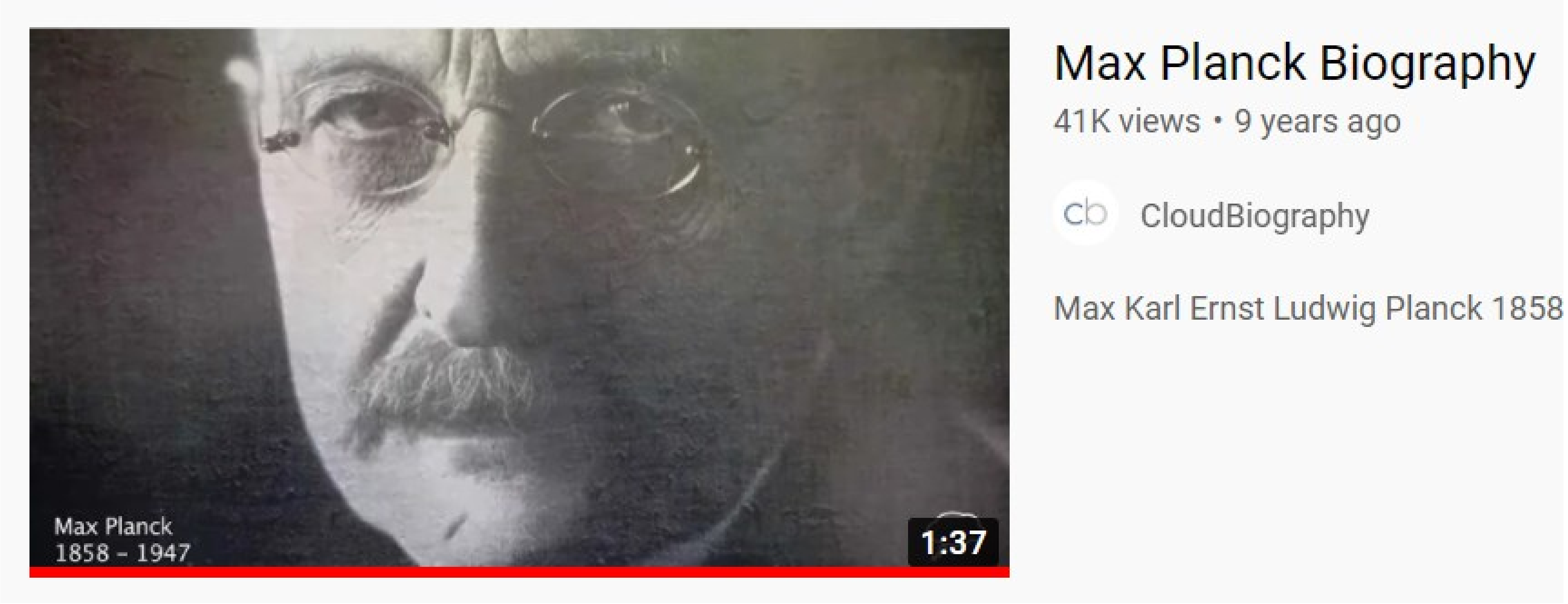
b. Kmax unchanged, # emitted electrons unchanged

c. Kmax is 4x, # emitted electrons unchanged

d. Kmax unchanged , # emitted electrons is 4x

e. Kmax is 2x , # emitted electrons unchanged

What is Planck's revolutionary discovery?



https://www.youtube.com/watch?v=R9sMxEnbha0

Origin of Plank's Constant | Birth of Quantum Mechanics | PHYSICA

233K views • 1 year ago

PHYSICA

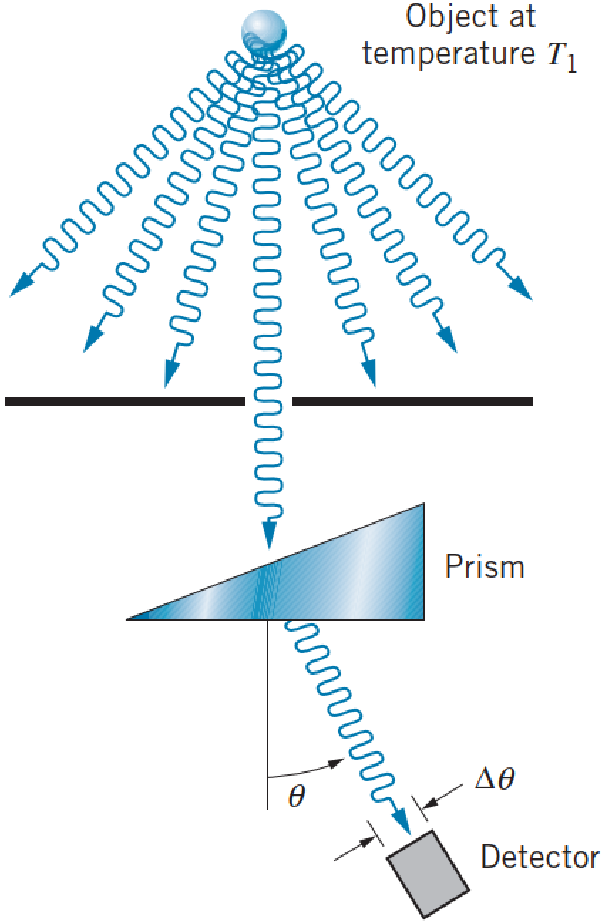
Physica #Planks #**constant** #**Origin** #PHYSICA An Initiative by a group of IIT Roorkee Scholars. Providing Fre

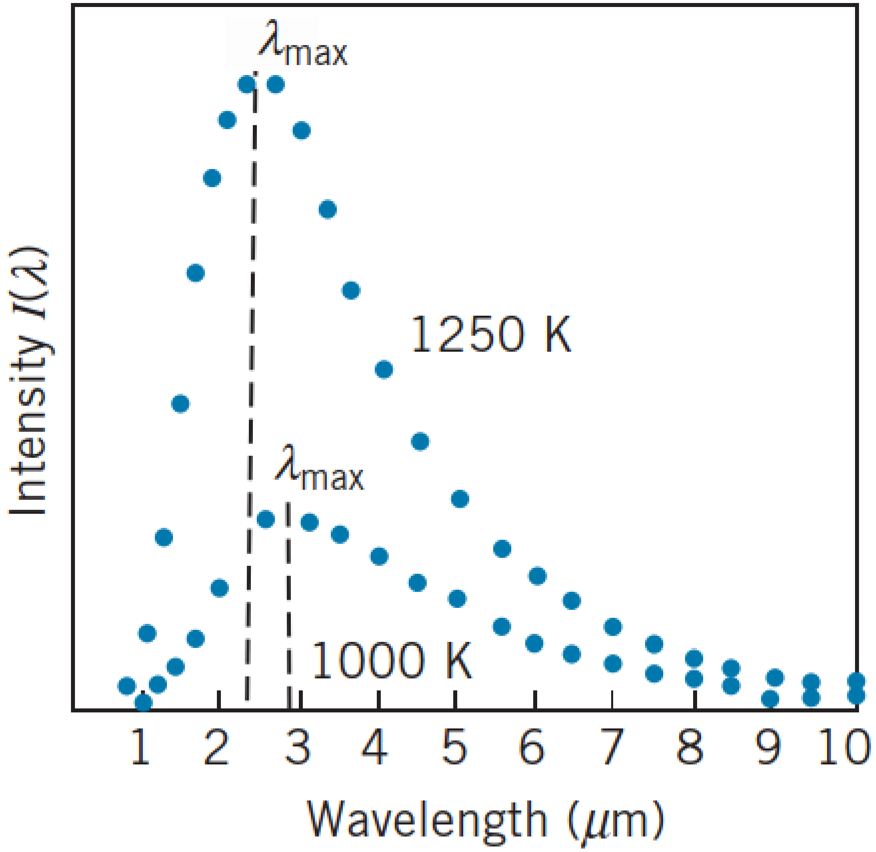
https://www.youtube.com/watch?v=2dgau46ZCqs

Heated objects emit light

Which one is the hottest?

Spectrum of thermal radiation I(λ)

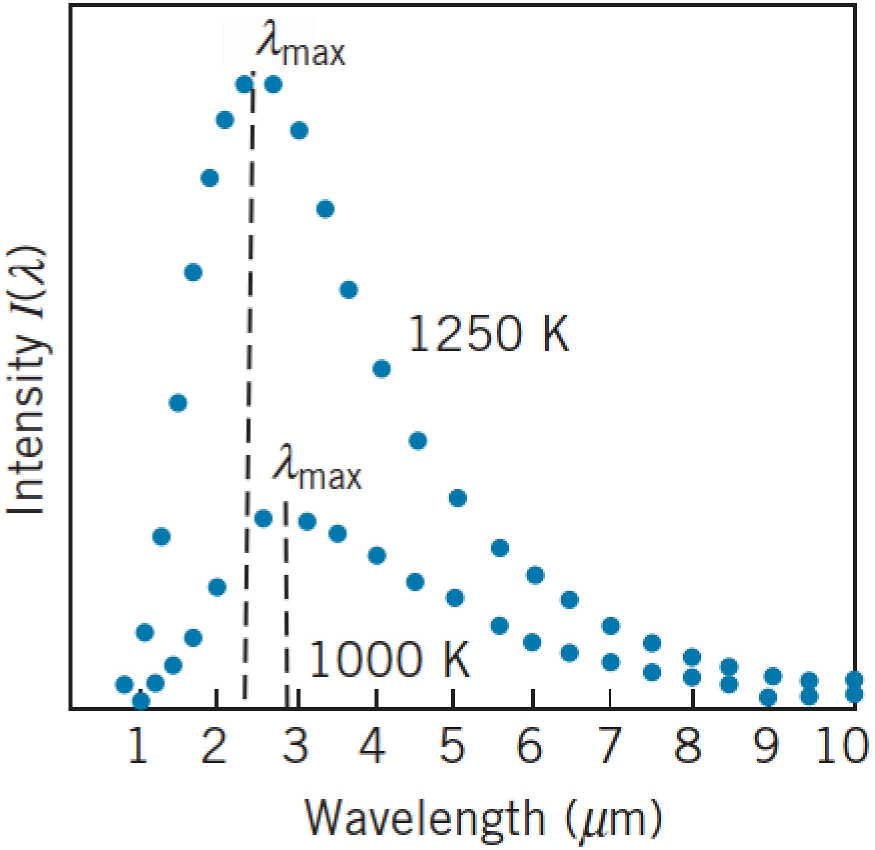




Two interesting observations here.
Can you find them?

Intensity: energy per unit time per unit area

Spectrum of thermal radiation I(λ)



Stefan-Boltzmann Law

Total intensity I= σT^4

σ = $5.67 \times 10^{-8} W m^{-2} K^{-4}$ Stefan-Boltzmann constant

Wien's Displacement Law:

λmax T = $2.898 \times 10^{-3}\, m \cdot K$

Spectrum of thermal radiation I(λ)

Use the approximation $\lambda max\ T = 3 \times 10^{-3}\, m \cdot K$ to estimate

λ_{max}

Sun: T = 6000K — 500nm

Room: T = 300K — 10,000nm — thermal vision

Universe: T = 3K — 1mm

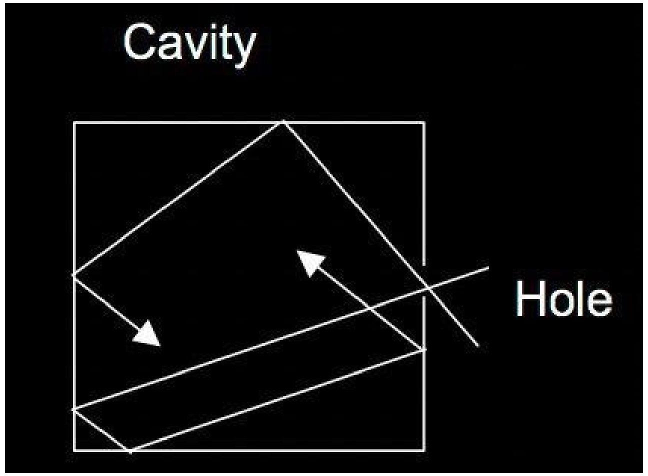
Black body radiation

Black body: An idealized object which absorbs all incident EM radiation and emits non directly. (A simplification so that calculations will be independent of the material property)

Idealized black body: a small hole to a cavity

- Light bounces around inside and is unlikely to directly get out
- But some thermalized radiation will make it out
- When a black body reaches thermal equilibrium, the intensity

of the emitted EM radiation equals the absorbed radiation



Classical crisis for black body radiation

- The classical calculations of the spectral intensity ($dI/d\lambda$) of the emitted radiation versus EM wavelength, as a function of the temperature yields (details skipped):

Rayleigh-Jeans Law

$$\frac{dI}{d\lambda} = \frac{2\pi ckT}{\lambda^4}$$

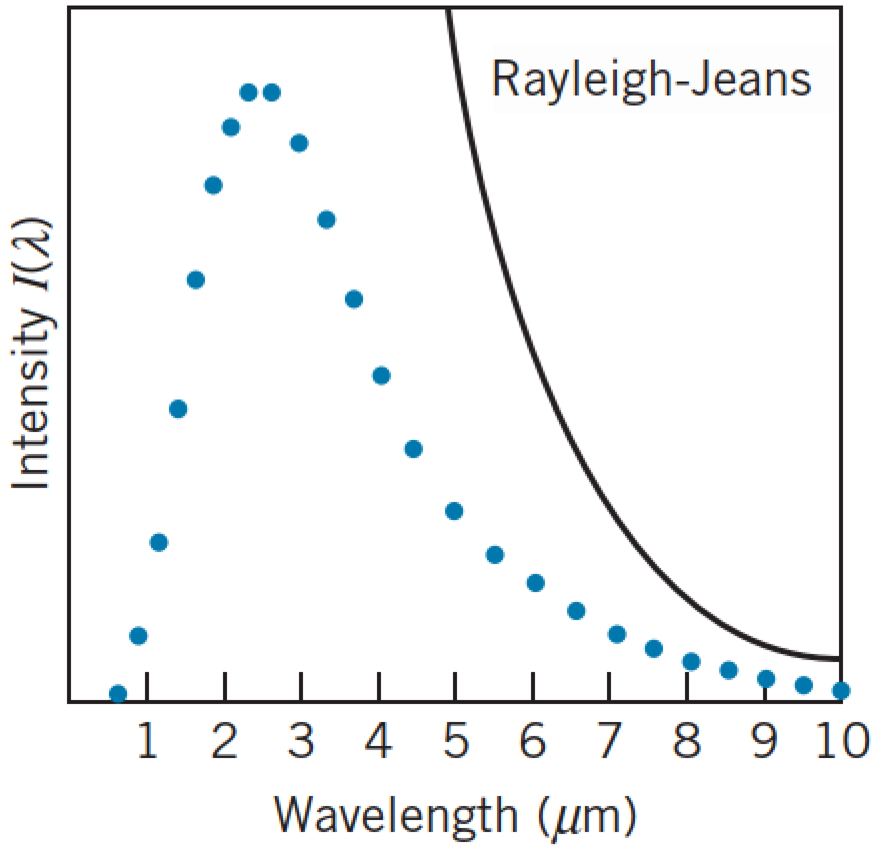
c = speed of light; Boltzmann constant $k = 1.38 \times 10^{-23} J/K$

Problem with Rayleigh-Jeans law: total intensity integrated over all wavelengths is infinite!

$$I = \int_0^\infty \frac{2\pi ckT}{\lambda^4} d\lambda \propto -\frac{1}{\lambda^3}\Big|_0^\infty \rightarrow \infty \ as\ \lambda \rightarrow 0$$

Obviously something wrong…

Ultraviolet catastrophe



- fits reasonably well for long wavelength light
- but strongly deviates for short wavelengths

Planck radiation formula

Max Planck derived the spectral intensity of black body radiation from a new assumption:
each atomic oscillator making up a black body can emit or absorb energy only in **interger multiples** of a **basic quantity of energy**

$$E_n = n\,\varepsilon \quad n = 1, 2, 3 \ldots$$

No longer continuous as in classical physics!

energy of each of the quanta is determined by the frequency

$$\varepsilon = h\,f$$

f is the frequency, Planck's constant $h = 6.6 \times 10^{-34}\ J \cdot s$

N_n: number of oscillators with energy E_n

$$N_n = A e^{-E_n/kT} = A e^{-n\varepsilon/kT}$$

$$N = \sum_{n=0}^{\infty} N_n = A \sum_{n=0}^{\infty} e^{-n\varepsilon/kT} = \frac{A}{1-e^{-\varepsilon/kT}}$$

$$A = N(1-e^{-\varepsilon/kT})$$

$$N_n = N(1-e^{-\varepsilon/kT})\, e^{-n\varepsilon/kT}$$

b/c $\sum_{n=0}^{\infty} e^{nx} = \frac{1}{1-e^{x}}$

Average energy Eav

$$E_{av} = \frac{1}{N}\sum_{n=0}^{\infty} N_n E_n = (1-e^{-\varepsilon/kT}) \sum_{n=0}^{\infty} (n\varepsilon) e^{-n\varepsilon/kT}$$

~~take~~ $\sum_{n=0}^{\infty} e^{nx} = \frac{1}{1-e^{x}}$, take derivative

$$d(\quad) = d(\quad)$$

$$d\left(\sum_{n=0}^{\infty} e^{nx}\right) = \sum_{n=0}^{\infty} n e^{nx} dx \ , \quad d\left(\frac{1}{1-e^{x}}\right) = -\frac{1}{(1-e^{x})^2} \cdot (-e^{x}) dx = \frac{e^{x}}{(1-e^{x})^2} dx$$

$$\sum_{n=0}^{\infty} n e^{nx} = \frac{e^{x}}{(1-e^{x})^2} \qquad \text{set } x = -\frac{\varepsilon}{kT}$$

$$\varepsilon = hf = \frac{hc}{\lambda}$$

$$E_{av}(\lambda) = (1-e^{x})\varepsilon \cdot \frac{e^{x}}{(1-e^{x})^2} = \frac{\varepsilon e^{x}}{1-e^{x}} = \frac{\varepsilon}{e^{-x}-1} = \frac{\varepsilon}{e^{\varepsilon/kT}-1} = \frac{hc/\lambda}{e^{hc/\lambda kT}-1}$$

I(λ) prediction

$$I(\lambda) = \frac{c}{4} u(\lambda) = \frac{c}{4} N(\lambda) E_{av}(\lambda) / V$$

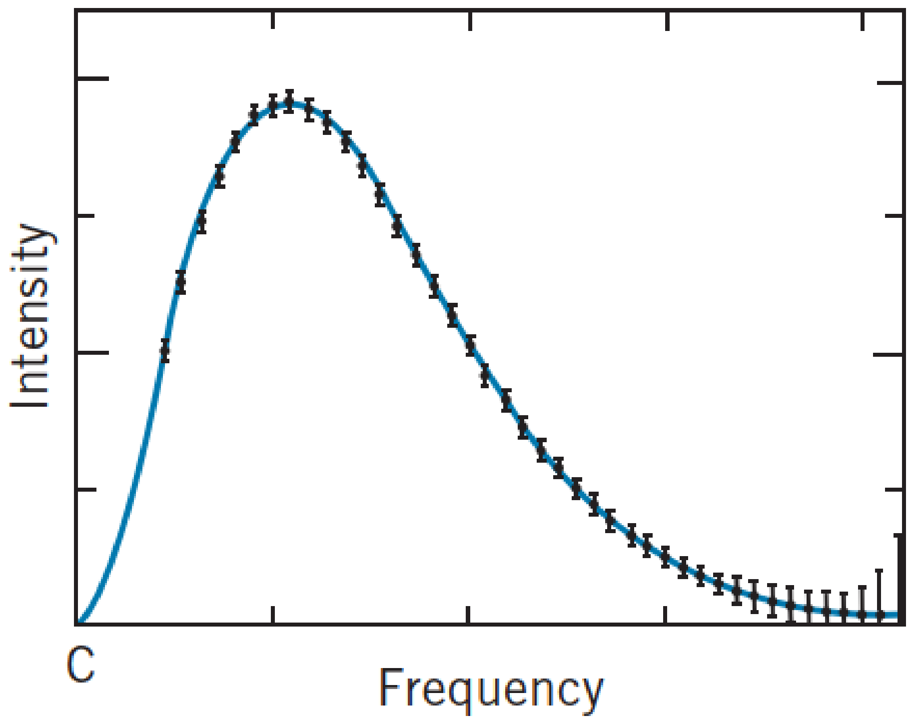
$u(\lambda)$ energy density

$$N(\lambda) d\lambda = \frac{8\pi V}{\lambda^4} d\lambda$$

# of standing waves btw λ & $\lambda + d\lambda$

$$I(\lambda) = \frac{c}{4} \cdot \frac{8\pi V}{\lambda^4} \cdot \frac{hc/\lambda}{e^{hc/\lambda kT} - 1} \cdot \frac{1}{V} = \boxed{\frac{2\pi h c^2}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1}}$$

Let’s test Planck’s prediction



- cosmic microwave background spectrum
- exactly fit (error bars enlarged 400x)
- estimated universe temperature 2.7K

Let's test Planck's prediction

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Stefan-Boltzmann Law

Total intensity I= σT^4

$\sigma = 5.67 \times 10^{-8} W m^{-2} K^{-4}$ Stefan-Boltzmann constant

- Find total power through integration (hint: use variable of $x = hc/\lambda kT$ for the integration)

$$I = \frac{2\pi^5 hc^2}{15}\left(\frac{k_B T}{hc}\right)^4 = \underbrace{5.67 \times 10^{-8}}\, T^4$$

Stefan-Boltzmann constant!

Let's test Planck's prediction

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Wien's Displacement Law:

$$\lambda_{max} T = 2.898 \times 10^{-3}\, m \cdot K$$

- Find peak wavelength from Planck's formula through setting the derivative to 0 (hint: use variable of $x = hc/\lambda kT$)

$$\lambda_{max} T = 0.002898 \text{ m} \cdot \text{K}$$

Wien's displacement law!

Let's test Planck's prediction

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

- At the long wavelength limit, $x = hc/\lambda kT \ll 1$, do a Taylor expansion

$$I_\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} \simeq \frac{2\pi hc^2}{\lambda^5} \frac{1}{hc/\lambda k_B T} = \frac{2\pi c k_B T}{\lambda^4}$$

Recovers Rayleigh-Jeans law!

- Short wavelength: $x = hc/\lambda kT \gg 1$

$$I_\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} \simeq \frac{2\pi hc^2}{\lambda^5} e^{-hc/\lambda k_B T}$$

Falls exponentially to zero Ultraviolet catastrophe solved!