Announcements

- Homework 8 is due next Wednesday, March 29.
- Exam 2 is on April 5 (schedule updated since the beginning of the semester).
- DRC test accommodation should be requested this week, if you haven't done it yet.
- Homework 9 is due April 3, Monday. Grader Mr. Sinha will finish grading HW9 before 3 pm on April 4.

Last time

- Probability densities (radial and angular)

Today’s class

- Spin

in-class quiz (5 min)

The angular probability density of state n=3, *l*=1, m_l=0 has its maximum value for θ

Hints: $\Theta(\theta) = \sqrt{\frac{3}{2}}\cos\theta$, $\Phi(\phi) = \frac{1}{\sqrt{2\pi}}$

A. 0 or π

B. π/2

C. π/4 or 3π/4

D. Not sufficient to determine.

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$$P(\theta, \phi) = |\Theta(\theta)\Phi(\phi)|^2 = \frac{3}{4\pi}\cos^2\theta$$

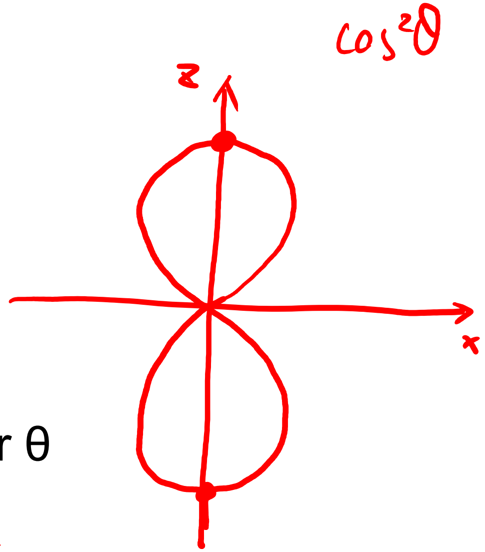
$$\frac{dP}{d\theta} = \frac{3}{4\pi}(-2\cos\theta\sin\theta) = 0$$

$\cos\theta = 0$, $\theta = \frac{\pi}{2}$

$\sin\theta = 0$, $\boxed{\theta = 0 \text{ or } \pi}$

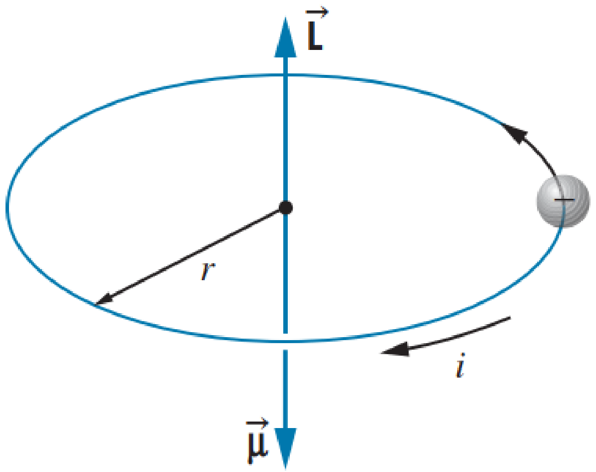
$$\frac{d^2P}{d\theta^2} = \frac{3}{2\pi}(1 - 2\cos^2\theta)$$

$\sin\theta = 0$, $\cos^2\theta = 1$, $\frac{d^2P}{d\theta^2} = -\frac{3}{2\pi} < 0$. maximum



$$\underline{P(r)\,dr = r^2 |R(r)|^2\,dr}$$

Orbital magnetic dipole moment



$$T = \frac{2\pi r}{v} = \frac{2\pi r m}{p}$$

$$\mu = iA = \frac{q}{T} A = \frac{qp}{2\pi r m} \cdot \pi r^2 = \frac{qpr}{2m} = \frac{qL}{2m} \qquad (L = rp)$$

$$\vec{\mu} = -\frac{e}{2m} \vec{L}$$

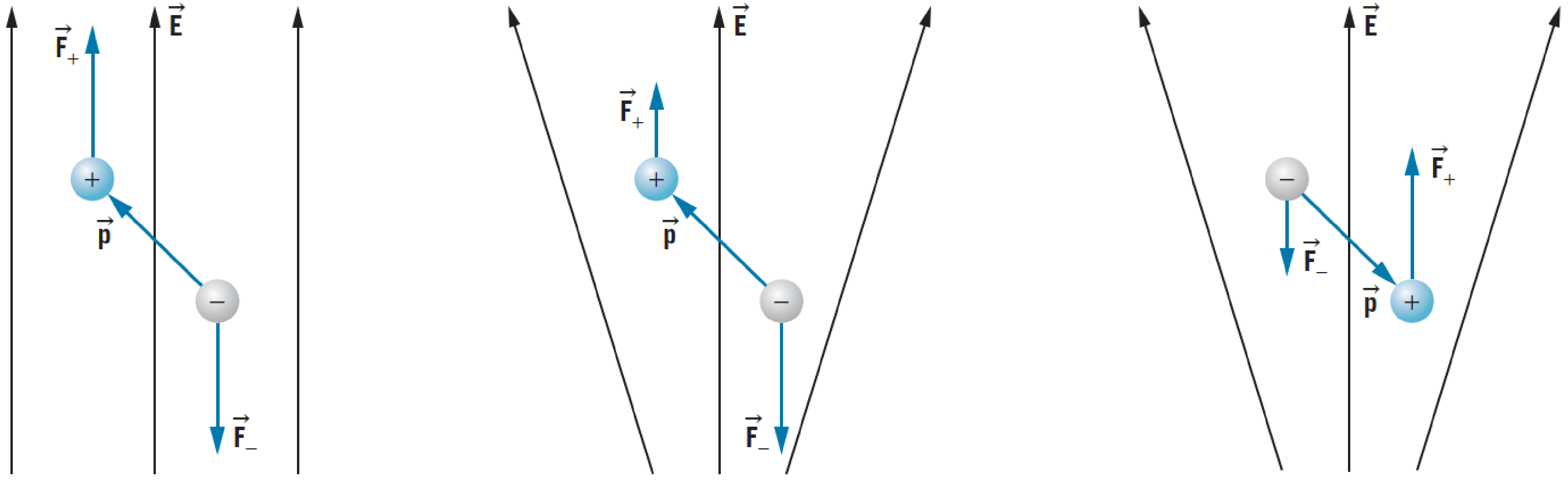
$$\mu_{L_z} = -\frac{e}{2m} L_z = -\frac{e}{2m} m_l \hbar = -\frac{e\hbar}{2m} m_l = -\mu_B m_l$$

$$\mu_B \equiv \frac{e\hbar}{2m} \quad \text{Bohr magneton}$$

$$= 9.274 \times 10^{-24} \text{ J/T}$$

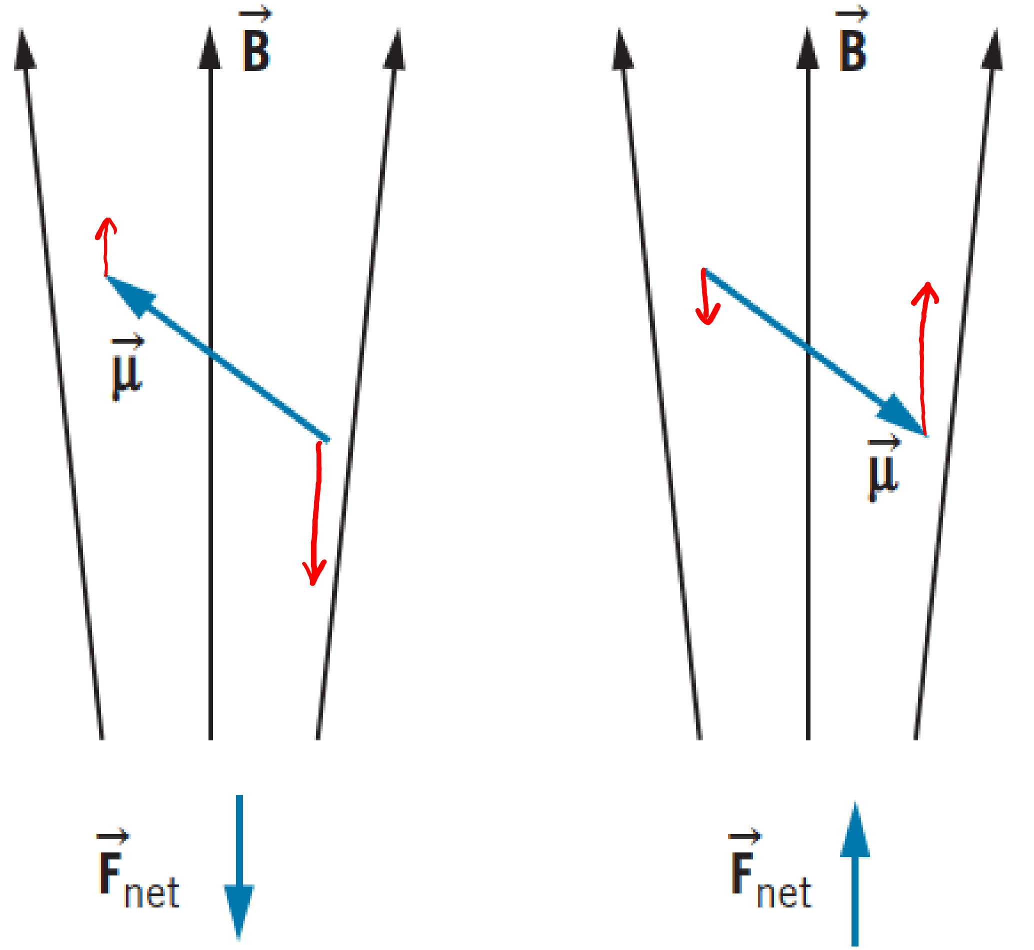
Tesla

Dipole in a non-uniform field



$F_{net} = 0$ $F_{net} \downarrow$ $F_{net} \uparrow$

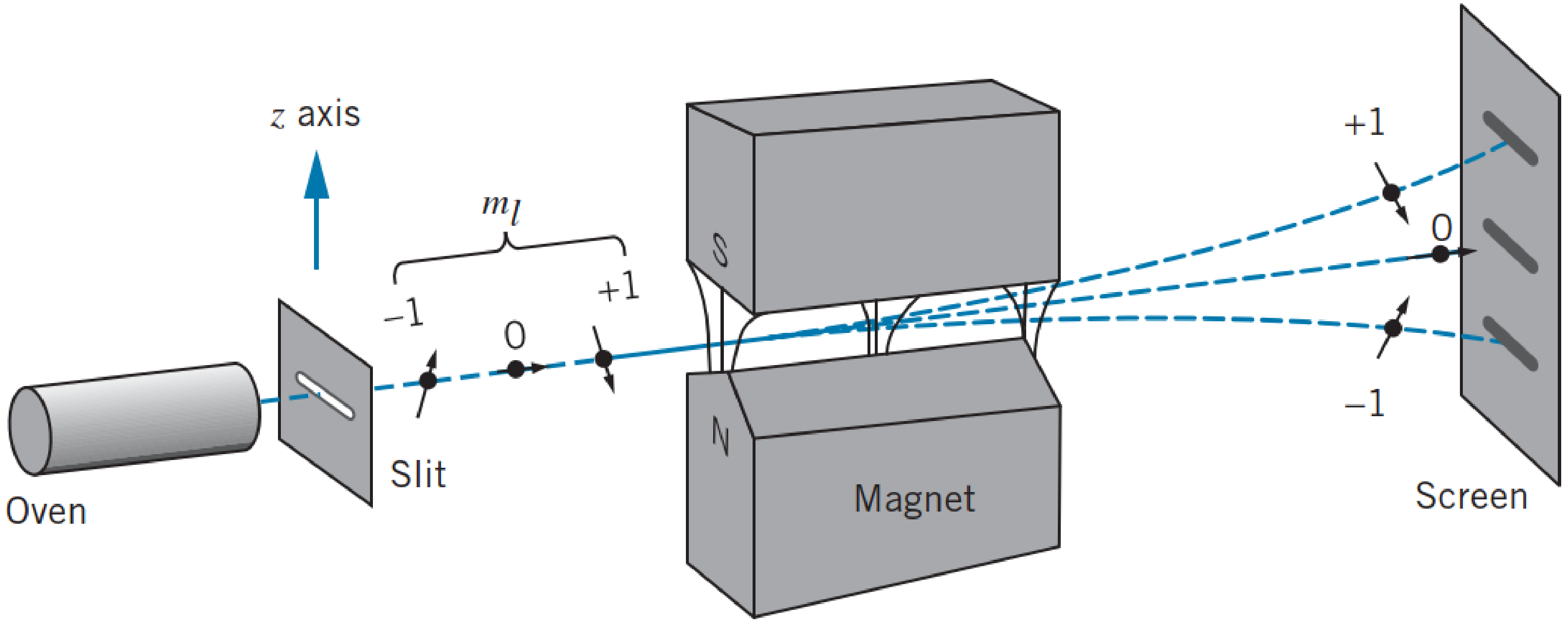
Dipole in a non-uniform field



z

$$\vec{F} = \vec{\mu} \cdot \frac{d\vec{B}}{dz} \hat{z}$$

The Stern-Gerlach Experiment



Predictions:

1. $2l+1$ positions – number of different m_l values.

(2l+1) positions → all possible m_l values.
$m_l = 0, \pm 1, \cdots, \pm l$

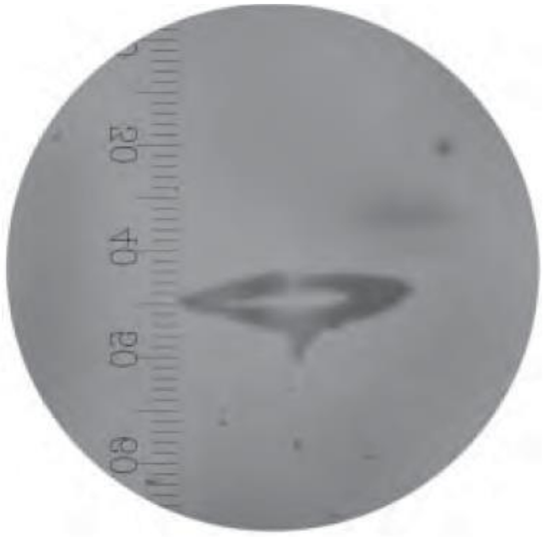
2. no deflection for $l = 0$ (i.e. m_l =0).

$l=0$, $m_l = 0$

The Stern-Gerlach Experiment

Field off

Field on



Observations:

1. the first conclusive evidence of spatial quantization
- atomic magnetic moments can take only certain discrete orientations in space

2. Silver, l=0, not ONE position as (2l+1), but split into TWO.
- Introduce “spin” - intrinsic angular momentum (2s+1) and set s=1/2.


Photo from the Nobel Foundation archive.

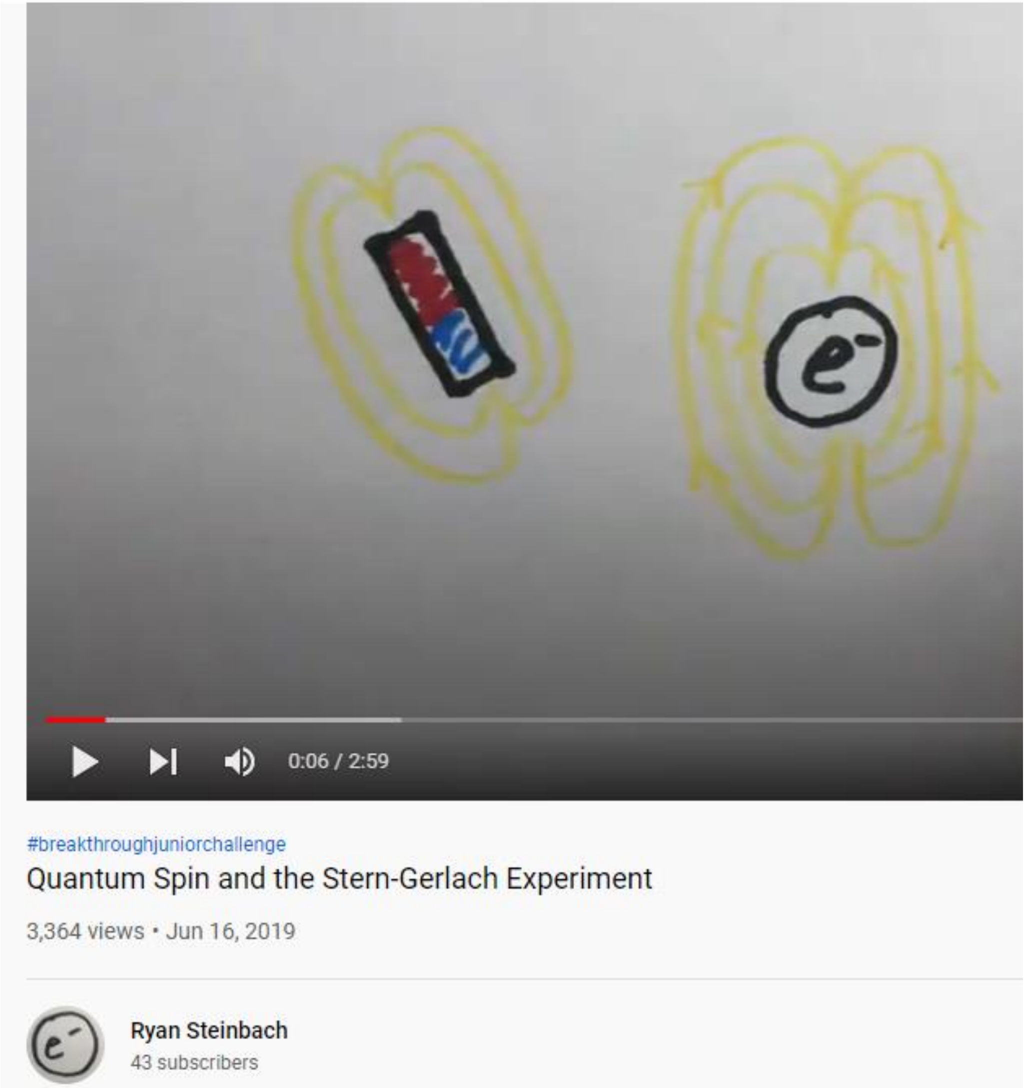
Otto Stern
The Nobel Prize in Physics 1943

Born: 17 February 1888, Sorau, Germany (now Zory, Poland)

Died: 17 August 1969, Berkeley, CA, USA

Affiliation at the time of the award: Carnegie Institute of Technology, Pittsburgh, PA, USA

Prize motivation: “for his contribution to the development of the molecular ray method and his discovery of the magnetic moment of the proton” https://www.youtube.com/watch?v=LGQdoK7OSWk



Spin: Intrinsic Angular Momentum

- **Spin** = *intrinsic angular momentum* of a particle. It is *not* a spinning ball of charge. it's an internal QM property with no classical analog.

like l and m_l, the spin state of a particle is described by s and m_s

$|\vec{L}| = \sqrt{l(l+1)}\,\hbar$, $L_z = m_l \hbar$, $m_l = -l, -l+1, \ldots, 0, \ldots, l-1, l$

$|\vec{S}| = \sqrt{s(s+1)}\,\hbar$, $S_z = m_s \hbar$, $m_s = -s, -s+1, \ldots, s-1, s$

total of $2s+1$ states

difference: l, m_l only integer numbers

s, m_s integer or half-integer

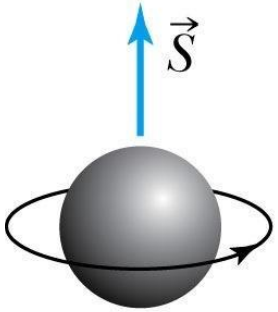
e^- : spin $s = \frac{1}{2}$, because in the S-G experiments, we only observed two states

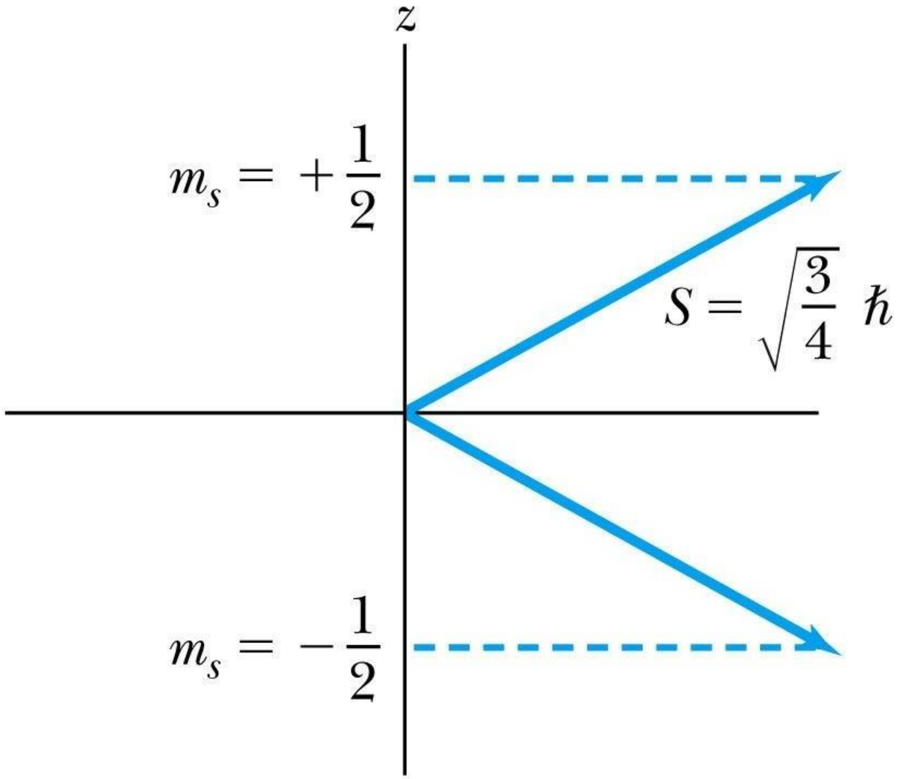
Two Spin States for s = ½

$|\vec{S}| = \sqrt{s(s+1)}\,\hbar = \sqrt{\frac{1}{2}\cdot\frac{3}{2}}\,\hbar = \sqrt{\frac{3}{4}}\,\hbar$

$S_s = m_s\hbar = -\frac{1}{2}\hbar$ or $\frac{1}{2}\hbar$

$m_s = -\frac{1}{2}, \frac{1}{2}$





Spin examples

- Examples
  - Electrons, positrons, protons, neutrons, neutrinos, quarks are spin ½
  - Photon is spin 1
  - π, K mesons are spin 0
  - Atoms are half-integral spin if $N_p + N_n + N_e$ is odd
  - Hydrogen atom has spin 0 or 1, depending on spin alignment

Four quantum numbers (n, l, m_l, m_s)

$\pm\frac{1}{2}$

- n : principle quantum number , n = 1, 2,…
- l : orbital angular momentum quantum number: $0 \le l \le n - 1$
- m_l: magnetic quantum number, $|m_l| \le l$
- m_s: spin quantum number

degeneracy $\quad n^2 \quad (n, l, m_l)$

$2n^2 \quad (n, l, m_l, \frac{1}{2})$

$(n, l, m_l, -\frac{1}{2})$

Spectroscopic notation

l =	0	1	2	3	4	5	6
	s	p	d	f	g	h	i

Selection rule $\Delta l = \pm 1$ is most likely.