Cavity design for a cosmic axion detector

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We discuss cavity tuning schemes for cosmic axion searches and strategies to extend such searches to high frequency without sacrifice in volume and hence signal-to-noise. Particular regard is given to the difficulties that arise from the tendency of the cavity mode to localize in a small region of the cavity and from the fact that the modes may become relatively dense in frequency space. Computer simulations of cavity performance have been carried out in order to optimize the cosmic axion search rate for a given magnet size and field.

INTRODUCTION

The axion is a hypothetical particle which has been postulated to explain, within the framework of the standard model of particle interactions, why parity P and the product CP of charge conjugation with parity are conserved by the strong interactions.[1,2] In addition to its significance for current theories of elementary particles, the axion may also be of cosmological importance. It has been shown that if the axion mass m_a is of order 10^{-5} eV, the axion may be the dominant matter in the universe.[3] Because axions are nonrelativistic from the moment (at about 1-GeV temperature) of their creation in the early universe, they are an excellent candidate[3,4] for constituting the dark matter that appears to be clustered in halos around galaxies.

Over five years ago, it was pointed out that axions can be searched for by stimulating their conversion to photons in a strong magnetic field.[5] In particular, an electromagnetic cavity permeated by a strong magnetic field can be used to detect galactic halo axions. Consider a cylindrical electromagnetic cavity of arbitrary cross-sectional shape, permeated by a large static, approximately homogeneous, longitudinal magnetic field $\mathbf{B} = B_0\hat{z}$. When the frequency $\omega = 2\pi f$ of an appropriate cavity mode equals $m_a[1 + O(10^{-6})]$, galactic halo axions can convert to quanta of excitation (photons) of that cavity mode. The range of $O(10^{-6})$ occurs because galactic halo axions would have velocities β of order 10^{-3} and hence their energies have a spread of order 10^{-6} above the axion mass. Only the TM_{nl0} modes couple in the limit where the cavity is much smaller in size than the de Broglie wavelength $\lambda_a = 2\pi(\beta m_a)^{-1} \approx 2\pi 10^3 m_a^{-1}$ of the galactic halo axions. The power on resonance $\{\omega_{nl} = m_a[1 + O(10^{-6})]\}$ from axion→photon conversion into the TM_{nl0} mode is[5]

$$P_{nl} = 2\times 10^{-20}\ \mathrm{W}\left(\frac{V}{500\ l}\right)\left(\frac{B_0}{8\ \mathrm{T}}\right)^2 C_{nl} \times\left(\frac{\rho_a}{\frac{1}{2}10^{-24}\ \mathrm{g/cm^3}}\right)\left(\frac{m_a}{2\pi(3\ \mathrm{GHz})}\right)\min\left(\frac{Q_L}{Q_a},1\right), \quad (1)$$

where V is the volume of the cavity, ρ_a is the density of galactic halo axions on earth, Q_L is the quality factor of the TM_{nl0} mode, and $Q_a \equiv 10^6$ is the "quality factor" of the galactic halo axions, i.e., the ratio of their energy to their energy spread.

The mode dependent form factor C_{nl} is given by

$$C_{nl} = \frac{|\int d^3x\ \hat{\mathbf{z}}\cdot\mathbf{E}_{nl}(x)|^2}{V\int d^3x\ \epsilon(x)|\mathbf{E}_{nl}(x)|^2}, \quad (2)$$

where $\mathbf{E}_{nl}(x)$ is the electric field of the TM_{nl0} mode. The quantity Q_L which appears in Eq. (1) is the loaded quality factor of the cavity, given by

$$\frac{1}{Q_L} = \frac{1}{Q_w} + \frac{1}{Q_h}, \quad (3)$$

where $1/Q_w$ is the contribution due to absorption into the cavity walls and $1/Q_h$ is the contribution from the coupling hole. The maximum power that can be brought to the front end of the microwave receiver is $(Q_L/Q_h)P$ where P is given by Eq. (1).

Because the axion mass is only known in order of magnitude (at best), the cavity must be tunable and a large range of frequencies must be explored seeking the axion signal. Tuning methods are described in Sec. I. Using Eq. (1), one finds that to obtain a given signal-to-noise ratio s/n, the search rate is

$$\frac{df}{dt} = \frac{2.6\ \mathrm{GHz}}{\mathrm{year}}\left(\frac{3n}{s}\right)^2\left(\frac{V}{500\ l}\right)^2\left(\frac{B_0}{8\ \mathrm{T}}\right)^4 C^2\left(\frac{\rho_a}{\rho_{\mathrm{halo}}}\right)^2\left(\frac{20\ \mathrm{K}}{T_n}\right)^2\left(\frac{f}{3\ \mathrm{GHz}}\right)^2\begin{cases}\dfrac{Q_w}{Q_a} & \text{if } Q_w < 3Q_a\\[2mm] \dfrac{27}{4}\left(1-\dfrac{Q_a}{Q_w}\right)^2 & \text{if } Q_w > 3Q_a\end{cases}, \quad (4)$$

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where $\rho_{\text{halo}} \equiv (1/2)10^{-24}$ gr/cm^3 and T_n is the sum of the physical temperature of the cavity plus the noise temperature of the microwave receiver. For the sake of brevity, we have dropped the mode indices from $C, f,$ and Q. Equation (4) was derived assuming (1) that when $Q_L < Q_a$, i.e., when the cavity bandwidth is larger than the axion bandwidth, one uses the possibility of looking at Q_a/Q_L axion bandwidths simultaneously,[6] and (2) that Q_h has been adjusted so as to maximize the search rate. For $Q_w < 3Q_a$, the optimal[6] $Q_h = \frac{1}{2}Q_w$ (and hence $Q_L = \frac{1}{3}Q_w$) whereas for $Q_w > 3Q_a$, the optimal Q_h is such that $Q_L = Q_a$. As far as the other components of the detector go, we have measured noise temperatures as low as 3 K (in the 1.2–1.6-GHz range) in commercially available microwave amplifiers. The best possible quality factors attainable at present (using oxygen-free copper) are only of order 10^5 in the GHz range, compared to the $O(10^6)$ for Q_a.

As was mentioned above, the range of axion masses one wishes to explore is rather large. There is an upper limit on the axion mass due to the combined constraints from stellar evolution and from high-energy laboratory searches.[2] This limit has recently been much improved[7] using the supernova, SN1987a. The new limit is $m_a \lesssim 10^{-3}$ eV or $m_a \lesssim 10^{-4}$ eV depending upon the details of the analysis.[7] Because of cosmological uncertainties which are hard to eliminate, the range of axion masses for which axions may be a viable dark matter candidate is large. It easily includes

$$2\pi(242\ \text{MHz}) = 10^{-6}\ \text{eV} \lesssim m_a \lesssim 10^{-4}\ \text{eV} = 2\pi(24.2\ \text{GHz}). \quad (5)$$

The question arises how one goes about exploring such a large range of frequencies.

Equation (4) suggests that with presently available technology, a search for galactic halo axions requires a magnetic field of order 8 T over a volume of order 500 ℓ, at least. The most appropriate magnet type is a superconducting solenoid. The volume requirement is met if, for example, the magnet inner radius $R_0 = 40$ cm and its length $L = 100$ cm. The frequency of the lowest TM mode (TM_{010}) of a cylindrical cavity of those dimensions is $f \approx 0.29$ GHz. This resonant frequency can be lowered somewhat through the insertion of dielectric rods into the cavity (see below). The $R_0 = 40$ cm, $L = 100$ cm, $B_0 = 8$ T magnet appears therefore adequate for exploring the low frequency end of the range [Eq. (5)]. How does one explore the higher frequencies? This is the question this paper attempts to answer. Specifically: (1) what is the best cavity design for exploring the higher frequencies? and (2) how large a frequency range can be explored? Equation (4) shows that C^2Q is the quantity that cavity design should optimize. In our discussions below, the values of Q quoted always refer to the $L = \infty$ case unless there is an explicit statement to the contrary. Indeed, in most of the situations of interest to us, the losses in the end caps are small compared to the losses in the side walls. In addition, when plots of C^2Q versus frequency are given (Figs. 3, 4, 6, 7, and 12) the frequency dependency of the skin depth is never included. Instead, a dimensionless quantity, $(C^2Q)_{\text{normalized}}$, is shown, normalized so that $C^2Q = 1$ for the empty cylindrical cavity without metal posts or dielectric inserts. This makes it possible to use our results for any detector size and surface conductivity by applying the formula

$$(C^2Q)_{\text{cavity}} = (C^2Q)_{\text{empty}}\,(C^2Q)_{\text{normalized}} = \frac{R}{\delta}(0.69)^2\ (C^2Q)_{\text{normalized}}. \quad (6)$$

Here, $(C^2Q)_{\text{cavity}}$ is the actual value of C^2Q of a cylindrical cavity of radius R containing certain metal posts and dielectric inserts in the limit where there are no dielectric losses and where the losses in the end caps can be neglected ($L = \infty$). For the lowest TM mode of an empty cylindrical cavity[5] the value of the form factor C is 0.69. δ is the skin-depth of the metal surfaces involved (cylinder and posts), assuming they all have the same surface conductivity.

In Sec. I, we will discuss different methods to tune a single cylindrical cavity. This will give us the opportunity to introduce the two main difficulties that present themselves in practical cavity design: resonance mode crossing and mode localization. In Sec. II, we attempt to determine the best overall design strategy for extending the axion search to high frequencies. In Sec. III, we summarize our conclusions.

I. CAVITY TUNING

The Brookhaven-Rochester-Fermilab[8] and Florida[9] pilot axion search experiments have been tuning their cavities by insertion of a dielectric rod. This tuning method was proposed and developed by the Brookhaven-Rochester-Fermilab collaboration. Although it is adequate for the purposes of these pilot experiments, one must recognize the limitations of the method when designating more ambitious axion detectors. These limitations are due to the phenomenon of mode localization. We distinguish two types of mode localization: transverse and longitudinal. The latter type creates the more severe problems. It is associated with a compromise of the longitudinal translational invariance of the cavity. For example, Fig. 1 shows a cylindrical cavity with a partially inserted dielectric rod. The fact that the rod is partially inserted breaks the translational invariance of the system. When the dielectric rod is fully inserted the translational symmetry is recovered. Let us briefly postpone further discussion of longitudinal mode localization and consider first cavities with fully inserted dielectric rods. They can still be plagued by the lesser problem of transverse mode localization.

Figure 2 shows $E_z(\rho)$ both for an empty cavity and for a cavity with a fully inserted dielectric rod. (z, ρ, θ) are cylindrical coordinates. In the case shown in Fig. 2, where $\epsilon = 10$ and the ratio of the radius of the dielectric rod to the radius of

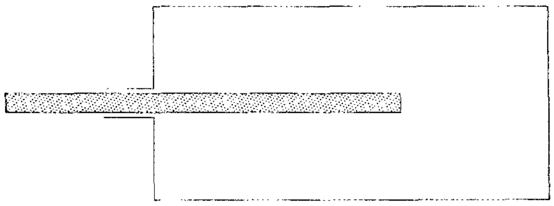

FIG. 1. Cylindrical cavity with a centrally placed partially inserted dielectric rod.

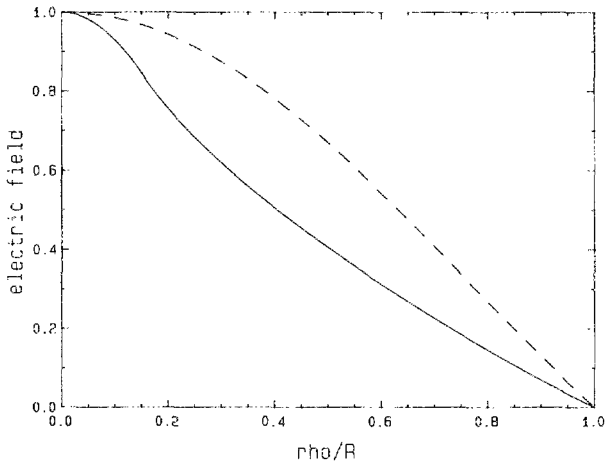

FIG. 2. Radial dependence of the longitudinal component of the electric field in the lowest TM mode of a cylindrical cavity with a fully inserted centrally placed dielectric rod with $\epsilon = 10$ and $r/R = 0.15$ (solid curve), and same for the empty cavity (dashed curve).

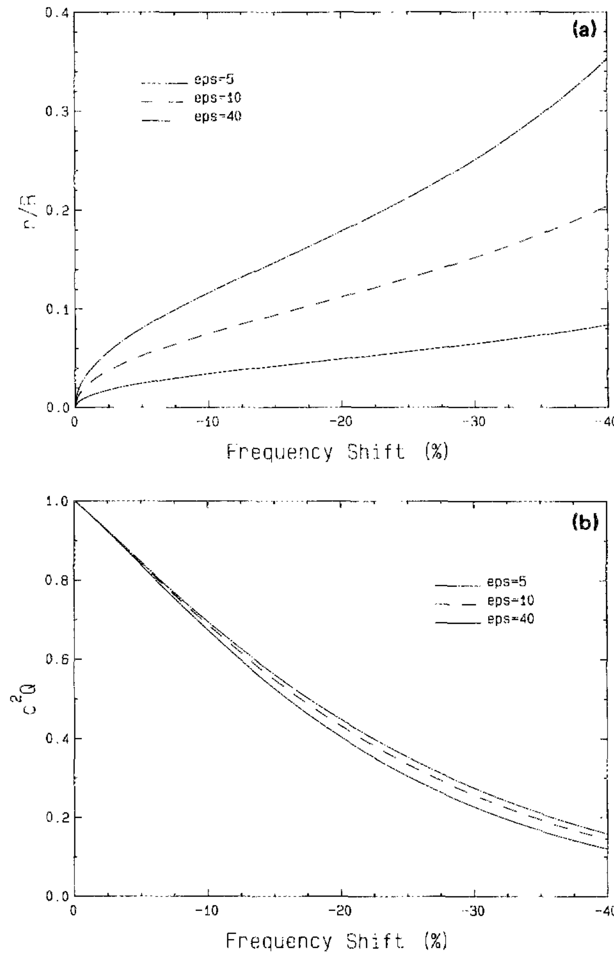

FIG. 3. Frequency shift (a) and C^2Q (b) as a function of rod size for a cylindrical cavity with a centrally placed fully inserted dielectric rod, for $\epsilon = 5$, 10, and 40.

the cavity $r/R = 0.15$, the shift in frequency was $\delta\omega/\omega = 30$ %. But there also was a reduction in C^2Q by a factor of four as compared to the empty cavity. Figure 2 shows that the mode tends to localize inside the dielectric; this in turn decreases C [see Eq. (2)] and hence the search rate which is proportional to C^2Q. Figure 3 shows C^2Q and r/R versus relative frequency shift $\delta\omega/\omega$ of a cylindrical cavity with a centrally placed fully inserted dielectric rod for $\epsilon = 5$, 10, and 40. For obvious reasons, one wishes to have the largest possible tuning range for each cavity that is constructed. However, Fig. 3(b) shows that, because of transverse mode localization, large $\delta\omega/\omega$ implies low C^2Q and hence a low search rate for given signal-to-noise.

Let us now return to longitudinal mode localization. Figure 4 shows how $\delta\omega/\omega$ and C^2Q vary as a function of the relative insertion depth l/L of the centrally placed dielectric rod for three different rods. Here, $R/L = 0.115$ and the contribution to $1/Q$ from the end plates is included. The figure shows that, whereas $\delta\omega/\omega$ is always a monotonicaly decreasing function of l/L, C^2Q first decreases and then increases with increasing l/L. When the dielectric rod is inserted half of the way, the value of C^2Q is lower than the already reduced value of C^2Q given in Fig. 3(b) for the fully inserted dielectric rod. The reasons for this is made clear by Fig. 5 which shows the electric field as a function of ρ and z for the mode of interest to us (TM_{010}) for $l = L$, $1/2\,L$, and $1/5\,L$. One notices that the mode tends to localize where the dielectric rod is, longitudinally as well as transversely. This tendency to localize in only part of the volume of the cavity can dramatically decrease C.

Consider a cavity which is almost translationally invariant. Its top part is slightly different from its bottom part. If both parts of the cavity were like its top part, the frequency of the lowest TM mode would be ω. If both parts of the cavity were like its bottom part, the frequency of the lowest TM mode would be $\omega + \delta\omega$. For example, the top part may have a dielectric rod inserted into it or may be slightly larger than the bottom part. It is easy to convince oneself that the lowest TM mode of such a cavity will tend to localize into its top part. The mode will fall off exponentially in the bottom part:

$$\psi \sim e^{-\kappa z}, \tag{7}$$

where z is the distance into the bottom part measured from the boundary between the two parts and

$$\kappa^2 = (\omega + \delta\omega)^2 - \omega^2 \simeq 2\omega\,\delta\omega\,. \tag{8}$$

If the half length $L/2$ of the cavity is large compared to the penetration length $1/\kappa$, the mode will essentially be confined to the top part. Hence we obtain the following general result: *if one wants to avoid large losses of C^2Q due to longitudinal localization of the mode, a certain minimum standard of longitudinal uniformity must be maintained.* This condition is expressed by

$$\frac{\delta\omega}{\omega} = \frac{\kappa^2}{2\omega^2} < \frac{2}{\omega^2 L^2} \tag{9}$$

if one requires C^2Q losses no worse than about 10%. If one allows C^2Q losses up to 50%, then the condition may be relaxed to $\delta\omega/\omega \lesssim 8/\omega^2 L^2$ (see Fig. 4). For a cylindrical cavity of radius R, the frequency of the lowest TM mode is given by $\omega_{010}^{TM} = 2.40/R$, and the corresponding maximum departure $\delta\omega/\omega$ from longitudinal uniformity is of order $(R/L)^2$. For example, for $L = 1$ m and $\omega = 2\pi(3$ GHz), $\delta\omega/\omega \lesssim 2.10^{-3}$ is required. Enforcing this constraint will sooner

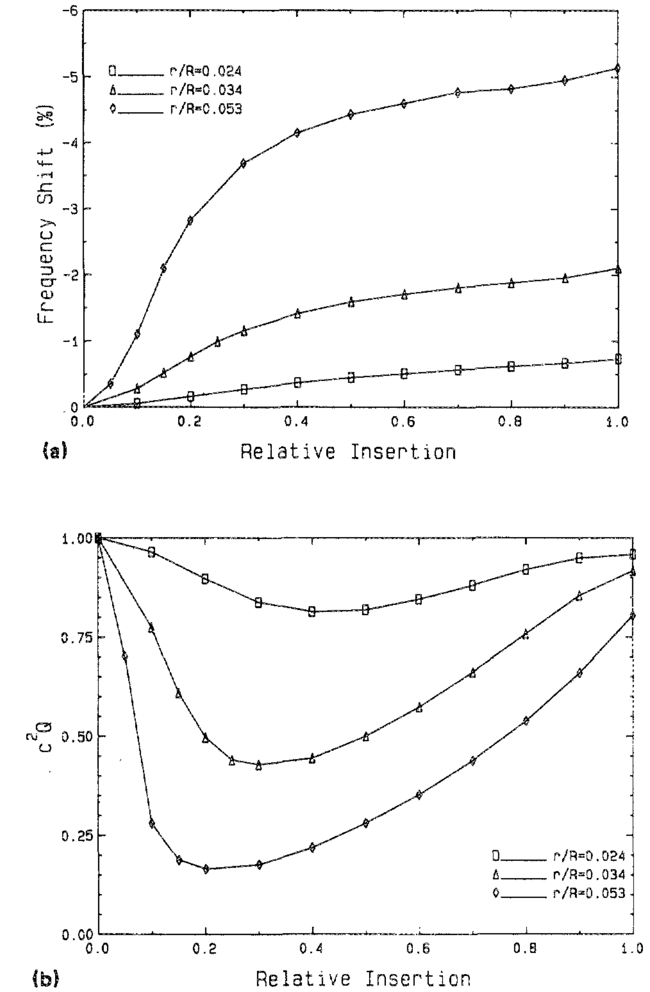

FIG. 4. Frequency shift (a) and C^2Q (b) as a function of the insertion depth of a centrally placed dielectric rod in a cylindrical cavity, for $\epsilon = 10$ and $r/R = 0.024$, 0.034, and 0.053. The aspect ratio of the cavity is $R/L = 0.115$. The data were obtained using the URMEL-T cavity mode computing program.

or later cause difficulties when ω is increased for fixed L.

The problem of longitudinal mode localization puts an upper limit, given by Eq. (9), on the tuning range if one inserts a single dielectric rod in the cavity. The problem may be avoided by inserting successively many thin rods. In this way, it is possible to never depart much from translational symmetry in the z direction. However, it may be rather difficult to accomplish this in practice. As an alternative, we propose to move the dielectric rod *sideways* inside the cavity, constantly keeping the translational symmetry in the z direction. Figure 6 shows how the frequency f and C^2Q change with the transverse displacement d of the rod for three different dielectric rods. The dip in C^2Q for intermediate frequencies is much less deep than when one tunes by inserting the rod (Fig. 4). We see that tuning by moving a dielectric rod sideways (constantly maintaining translational invariance in the z direction) avoids the problem of low C^2Q due to longitudinal mode localization. The tuning range is then limited only by the problem of low C^2Q due to transverse mode localization.

One can also tune the cavity by moving a metal post sideways inside the cavity, again enforcing translational symmetry in the z direction at all times [Eq. (9)]. Transverse motion does not require a complicated mechanical design. Instead of a lossy mechanical contact we left a small gap with large capacitance between post and cavity. This is to avoid distortions of the field distribution (due to lack of longitudinal invariance) and concomitant loss is C^2Q. Figure 7 shows how the frequency f and C^2Q change as a function of the transverse displacement d of a cylindrical metal post of radius r from the center of a cylindrical cavity of radius R for three values of r/R. It shows that this method allows relatively large tuning range without a heavy price in C^2Q, e.g., $\delta\omega/\omega = 35\%$ with $C^2Q_{\min} = 45\%$ of C^2Q_{empty}. We constructed several cavities tuned by the transverse motion of a metal or dielectric rod. They worked quite satisfac-

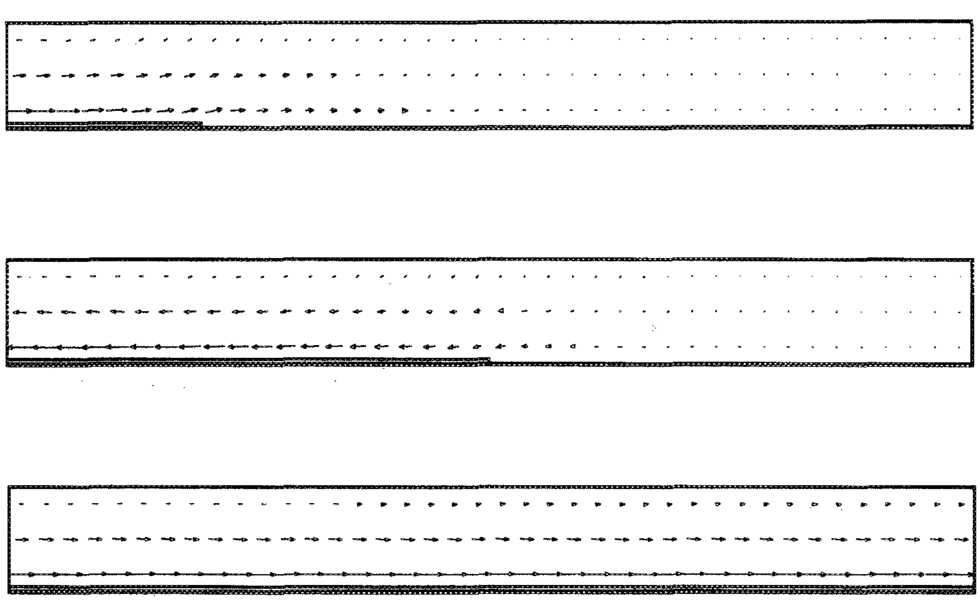

FIG. 5. Electric field configuration of the lowest TM mode of a cylindrical cavity containing a centrally placed dielectric rod for three different insertion depths. The data were generated by the URMEL-T program.

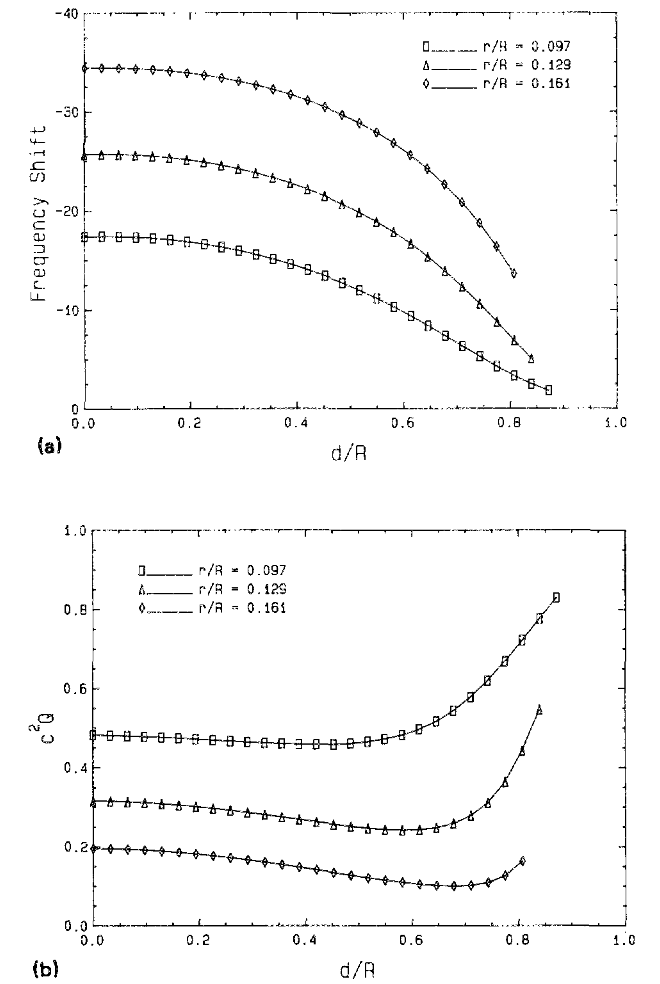

FIG. 6. Frequency shift (a) and C^2Q (b) vs transverse displacement of a fully inserted dielectric rod in a cylindrical cavity, for $\epsilon = 11$ and $r/R = 0.097$, 0.129, and 0.161.

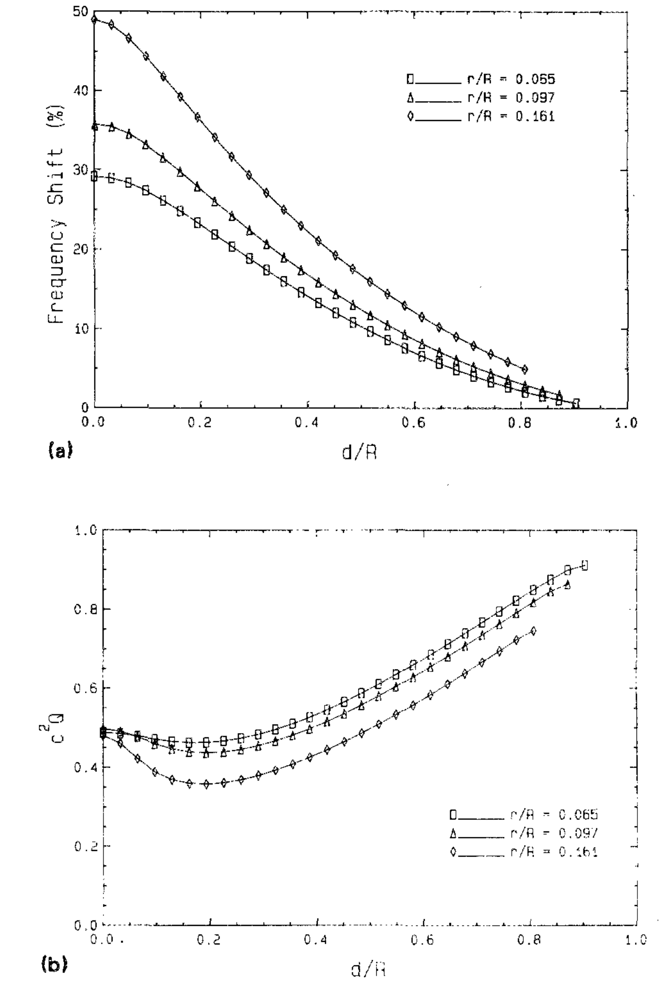

FIG. 7. Frequency shift (a) and C^2Q (b) vs transverse displacement of a fully inserted metal post in a cylindrical cavity, for $r/R = 0.065$, 0.097, and 0.161.

torily. Figure 8 shows the measured frequency of the lowest TM mode as a function of the displacement d of the metal post from the center of a cylindrical cavity of radius $R = 3.6$ cm and length $L = 15.2$ cm. The radius of the metal post is $r = 0.46$ cm. The figure also shows, as a function of d, the calculated frequencies of the lowest TM mode and of all the TE modes in the tuning range. In addition to the TM mode and the TE modes, there is, within this tuning range, a TEM mode at 3.95 GHz. There is relatively good agreement between the measured and calculated values of the lowest TM mode. However, in the vicinity of each mode crossing, the modes involved mix and repel each other (see Fig. 9) due to the presence of small asymmetries in any practical cavity. This mixing leads to holes in the cavity tuning range where the axion search cannot be carried out. The location of these holes changes from cavity to cavity however so that it is still possible to search all axion masses by changing cavities. The measured quality factor of the cavity with the moving metal post was everywhere within 30%–50% below the calculated value. The extra losses can be attributed to the slits in the top and bottom plates of the cavity that were used to implement the tuning mechanism. The presence of these slits was not included in the theoretical calculation.

The TM mode used in an axion search can become so crowded by TE and TEM modes that, because of mode mixing, Q becomes too degraded or the mode becomes too difficult to track. This is the second difficulty which must be faced in the design of electromagnetic cavities for cosmic axion searches although it appears that this is a less severe problem than the problems of transverse and longitudinal mode localization emphasized above. Consider a rectangular cavity of length L and transverse sizes a and b. The rectangular shape is chosen because it allows the discussion to be both simple and sufficiently general. The modes of this cavity have frequencies

$$\omega_{mnp} = \pi\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{L}\right)^2\right]^{1/2}. \quad (10)$$

For TM modes, $p = 0,1,2,...$ and $m,n = 1,2,3...$. For TE modes, $p = 1,2,3,...$ and $m,n = 0,1,2,...$ except $m = n = 0$. For a cavity that is longer than it is wide, the lowest TE modes are below the lowest TM mode. One easily finds that in the limit of large mode numbers, the density of TE or TM modes in frequency space

$$n(\omega) = \frac{dN}{d\omega} \cong \frac{4abL}{\pi^2}\,\omega^2. \quad (11)$$

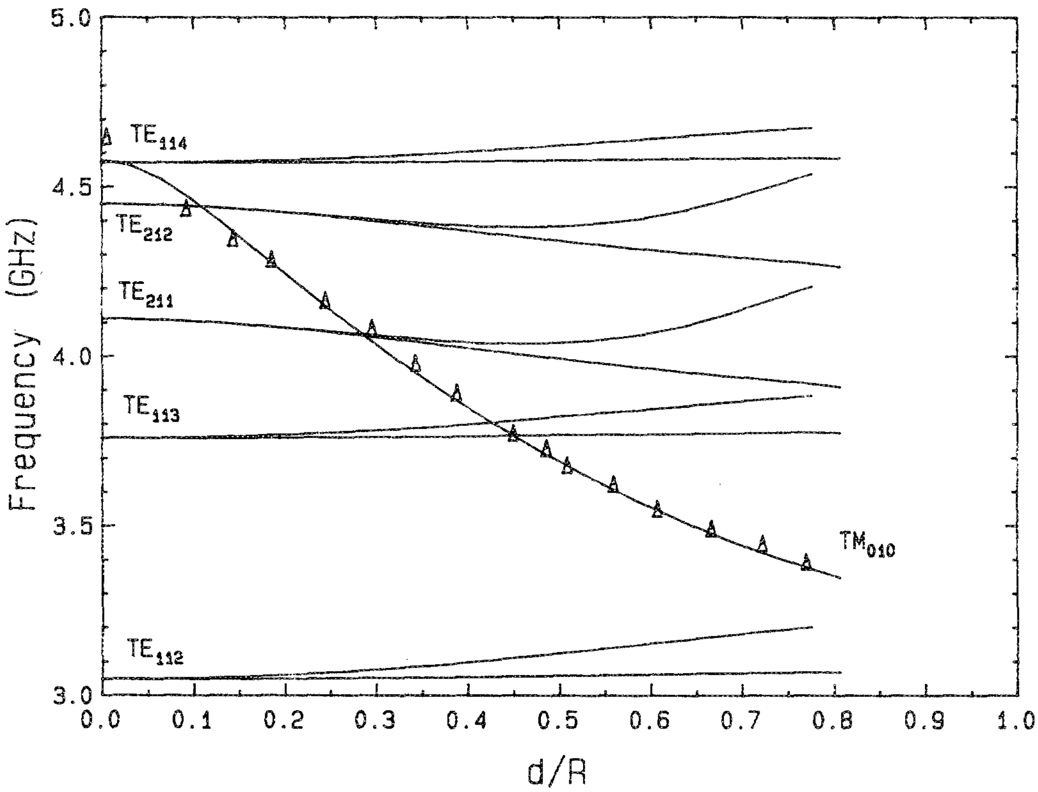

FIG. 8. Frequency vs transverse displacement of fully inserted metal post in a cylindrical cavity for $R = 3.6$ cm, $r = 0.46$ *cm*, and $L = 15.2$ cm. Triangles indicate the measured values for the TM_{010} mode.

If $a \sim b \ll L$, the density $n_0(\omega)$ of TE modes near the frequency $\omega \cong \sqrt{2}\pi/a$ of the lowest TM mode is of order

$$n_0(\omega) \cong 8L \,. \tag{12}$$

To avoid resonance crowding, one must require

$$n_0(\omega)\Delta\omega = n_0(\omega)\frac{\omega}{Q_L} \ll 1 \,. \tag{13}$$

For example, if $L = 1$ m and $Q_L \cong R/3\delta \cong 1.3 \times 10^5$ (0.3 GHz/f)$^{2/3}$ in the anomalous skin depth regime, we have

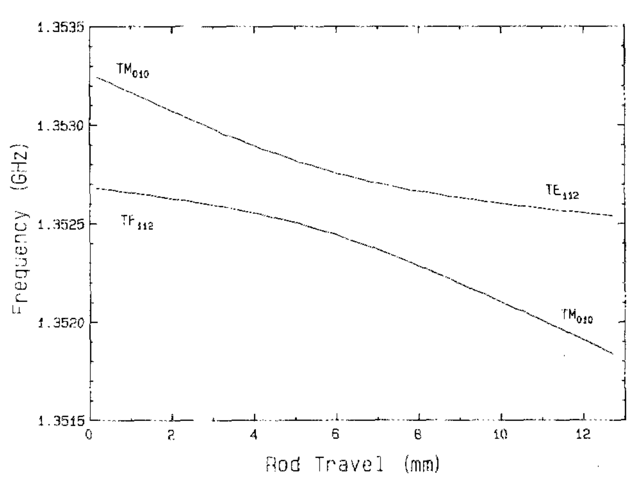

FIG. 9. Measured frequency of TM_{010} and TE_{112} modes vs tuning rod position.

$$n_0(\omega)\Delta\omega = 4.10^{-4}\left(\frac{f}{0.3\ \text{GHz}}\right)^{5/3} . \tag{14}$$

If we take $n(\omega)\,\Delta\omega < 0.02$ to be the condition that resonance crowding is not excessive, we find that the search can be extended only to frequencies of order 3 GHz in this particular case.

II. HIGH FREQUENCY LARGE VOLUME CAVITY DESIGN

We now return to the question posed in the introduction: assuming the availability of a large superconducing solenoid—e.g., $R_0 = 40$ cm, $L = 100$ cm, and $B_0 = 8$ T—how does one proceed to explore high-frequency ranges in a cosmic axion search? The following methods have been considered:

(1) One can decrease the radius R of the cylindrical cavity. Assuming B_0, T_n, and L fixed, one has $V \sim R^2 \sim f^{-2}$, $C \sim f^0$ and $Q_w \cong R/\delta \sim f^{-2/3}$ because δ goes like $f^{-1/3}$ in the low-temperature anomalous skin depth regime. Hence, using Eq. (4), we obtain

$$\frac{d\ln f}{df} \sim f^{-11/3}\, T_n^{-2}(f) \,. \tag{15}$$

To extend the search to higher frequencies, one wishes ideally that $d\ \ln f/df \sim f^{\alpha}$ with $\alpha \gtrsim 0$. Equation (15) tells us that method 1 is unsatisfactory. Of course, the main reason for this is the decrease in volume V. Most of the available volume inside the magnet is left unused. One can use the extra volume for an insert that increases the magnetic field but magnetic fields are limited from above to about 15 T.

(2) One can use the high frequency TM_{0n0} modes of a single cavity of maximum volume allowed by the magnet size. From Ref. 5 one finds that $C \sim f^{-2}$ and because the cavity radius does not change, $Q \sim f^{1/3}$. Hence

$$\frac{d \ln f}{df} \sim f^{-8/3} T_n^{-2}(f), \qquad (16)$$

which is better than method 1 but is still unsatisfactory. Also, the problem of resonance mode crossing is rather severe in method 2. Using Eq. (11) with $a = b \cong 1.8R$, $R = 40$ cm and $L = 100$ cm, taking $n(\omega)\,\Delta\omega = n(\omega)\,\omega/Q_L < 0.02$ to be the condition that mode crowding is not excessive and using $Q_L = 1.3 \times 10^5 (f/0.3\ \text{GHz})^{1/3}$ we find that the search can be extended only to about 1 GHz in method 2.

(3) Morris[10] proposed to increase the form factor C by the judicious insertion of dielectric materials into the cavity. As in method 2, the cavity volume is the maximum allowed by the magnet size and the higher TM modes are used to extend the axion search to high frequency. But now one inserts into the cavity longitudinal slabs of dielectric material whose shape, size, and position are carefully chosen to maximize C. The main idea [cf. Eq. (2)] is to have high ϵ material in the cavity wherever $E_{z,nl}(x,y)$ is negative and absence of such material wherever $E_{z,nl}(x,y)$ is positive. In this manner, one can obtain $C = 0(1)$ no matter how high the mode number. Assuming no dielectric losses, we have $Q_\omega \sim f^{+1/3}$ and hence

$$\frac{d \ln f}{dt} \sim f^{4/3} T_n^{-2}(f). \qquad (17)$$

which is very favorable for an extension of the axion search to high frequencies. On the unfavorable side, for this method to work, the dielectric material must be constantly modified in size and position as the frequency is shifted. This seems very difficult to do in practice. In addition, the mode crossing problem is certain to be at least as severe in this method as it is in method 2.

(4) One can insert metal posts into the cavity. Figure 10 shows, as an example, the cross-sectional view of a cylindrical cavity with four longitudinal, symmetrically placed metal posts. The radius of these posts and their displacement from the center of the cavity were chosen so as to maximize at a given frequency the value of C^2Q and hence of the search rate in the lowest TM mode. Figure 11 shows the profile of the lowest TM mode. Its frequency is more than double the frequency of the lowest TM mode of the empty cavity because of the "pinching" effect of the metal posts. The value of the form factor C, however, is $0(1)$. In this method, $V \sim f^0$, $c \sim f^0$, $Q_\omega \sim f^{-2/3}$ and hence

$$\frac{d \ln f}{dt} \sim f^{1/3} T_n^{-2}(f) \qquad (18)$$

which is acceptable for extending the axion search to high frequencies provided the noise temperature does not increase too rapidly with increasing f. This method may be attractive up to frequencies equal to two or three times the frequency f_0 of the fundamental TM mode of the cavity without metal posts. The problem which sets in then and which becomes increasingly intractable at higher frequencies is transverse mode localization. Figure 11 shows that the cavity with four metal posts whose cross section is shown in Fig. 10 has five cells, i.e., five regions in which the lowest TM mode bulges. To obtain $C = 0(1)$, it is necessary that the mode bulges approximately equally in each of the five cells. However, deviations of the radii and positions of the metal posts from their optimum values can cause the mode to localize in one of the cells. For example, Fig. 12 shows the lowest TM mode when the displacement of the four metal posts from the center has been increased by 11% from the optimal position. In Fig. 12, the mode is almost entirely localized inside the central cell. The resulting form factor is $C = 0.55$, which is considerably below the optimum value $C = 0.72$ corresponding to Fig. 11. To avoid transverse mode localization when the cavity is tuned, one must therefore tune each of the cells of the cavity simultaneously. The tuning can be carried out using any of the methods discussed in the previous section, with similar tuning ranges, but careful computer simulations will have to be carried out to insure that transverse mode localization is avoided at all frequencies of the tuning range. Clearly, it will become more and more difficult to do this as the number of cells increases. Figure 13 shows C^2Q versus frequency of optimized cavities with one and five metal posts. In each case, the radius and the displacement from the center of the n identical symmetrically placed metal posts ($n = 1,5$) were varied until maximum C^2Q was obtained at a given frequency. The curves for $n = 2,3$, and 4 lie for the most part below the ones for $n = 1$ and 5 and were omitted. Figure 13 also shows C^2Q for an optimized infinite triangular lattice. The lattice was optimized in the sense that the radius of the identical metal posts and the distance between them (i.e., the unit cell size) were varied till maximum C^2Q was obtained. The density of TE modes about the lowest TM mode of a cylindrical cavity with n metal posts is given in order of magnitude by Eq. (11) with $a = b \sim 1.8R$, i.e.,

$$n(\omega) = \frac{dN}{d\omega} \approx 50R^2Lf^2. \qquad (19)$$

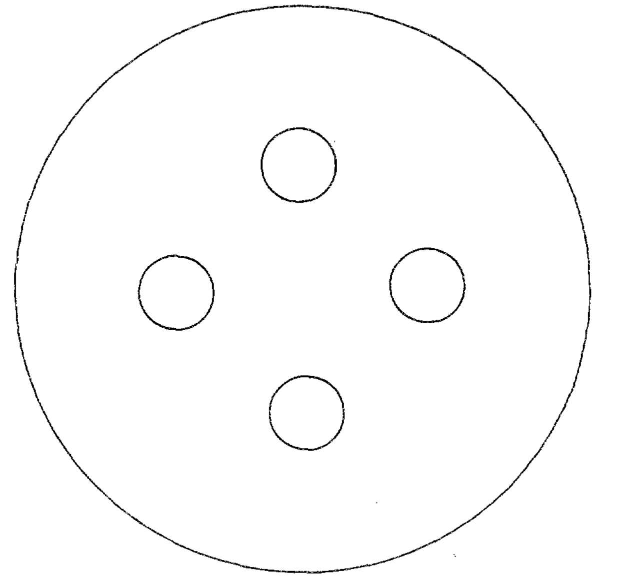

FIG. 10. Cross-sectional view of a cylindrical cavity with four metal posts.

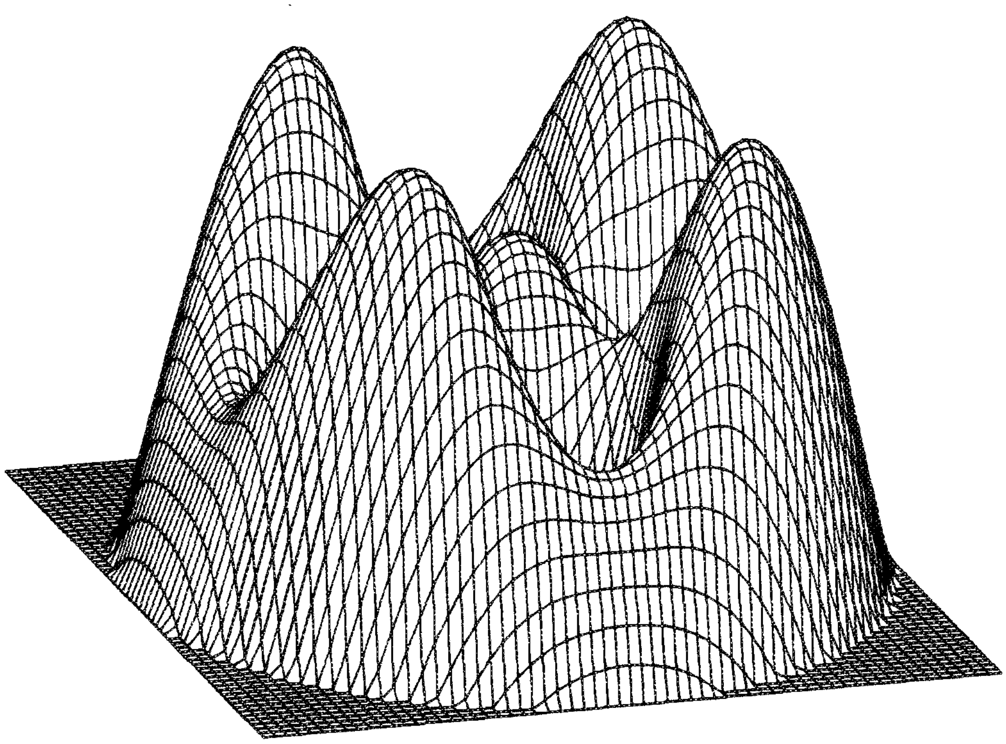

FIG. 11. Profile of the longitudinal electric field of the lowest TM mode of the cavity represented in Fig. 8.

This is because the frequencies of the TE modes are not changed to lowest order by the insertion of the metal posts, whereas those of the TM modes are pushed up. Figure 8 shows this in the case $n = 1$. In addition to the TE modes, there are n degenerate TEM modes at each frequency $\omega_p = \pi p/L$ for $p = 1,2,3...$. Clearly, the density of TE modes is much larger than that of TEM modes. For $L = 100$ cm and

$$Q_L = 1.3\times 10^5 \ (0.3 \ \mathrm{GHz}/f)^{2/3},$$

we have

$$n(\omega)\Delta\omega = n(\omega)\omega/Q_L \simeq 0.4\times 10^{-3}(f/0.3 \ \mathrm{GHz})^{11/3}.$$

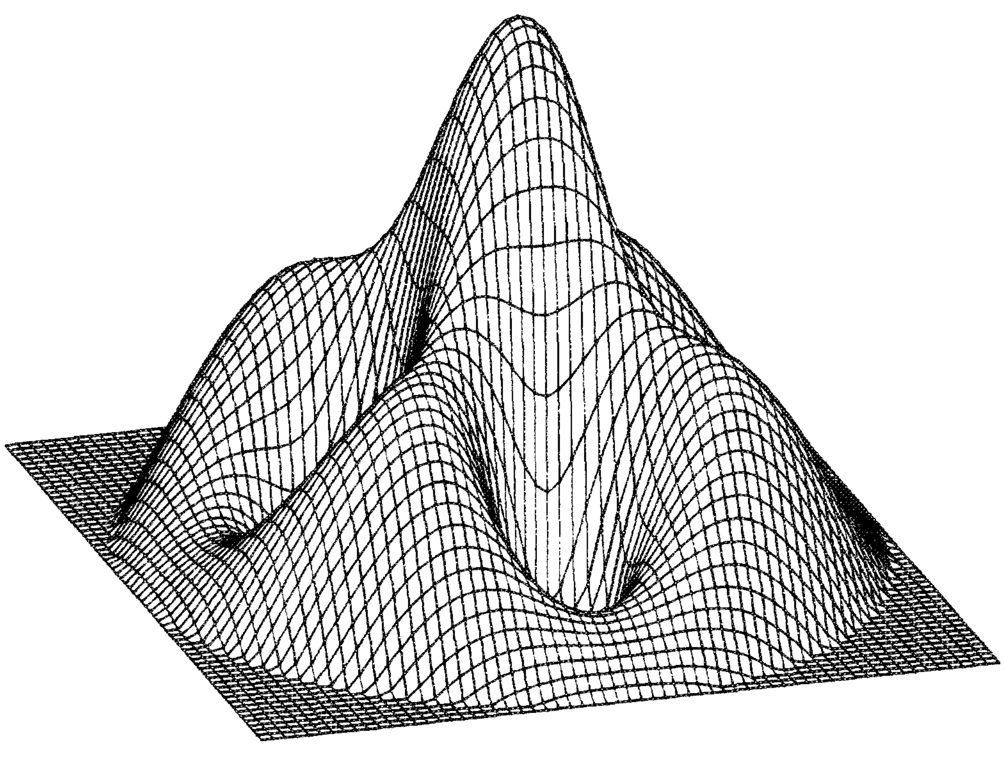

FIG. 12. Same as in Fig. 9 but with the metal posts' displacement from the center increased by 11%.

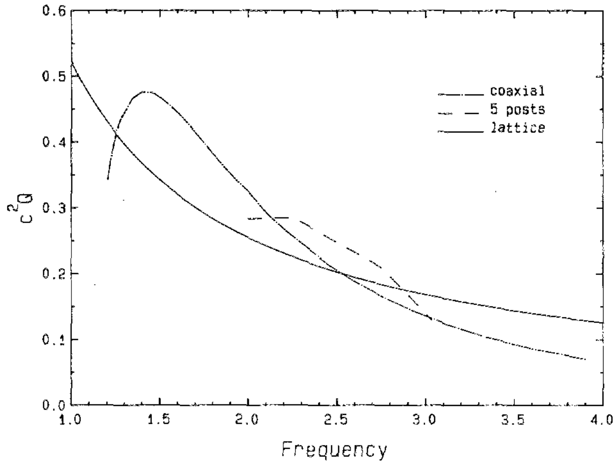

FIG. 13. C^2Q vs frequency of optimized cavities with one and five metal posts. Also shown is C^2Q for an optimized infinite triangular lattice.

If we take $n(\omega)\ \Delta\omega < 0.02$ to be the condition that mode crowding is not excessive, we find that the search can only be extended to about 0.8 GHz in method 4 for this particular example.

(5) The fifth scheme is less imaginative than most of the others but it may be the most practical. It consists simply of close-packing as many identical cavities as possible within the volume available inside the magnet. Each cavity is tuned independently (see Sec. I) and kept at the same frequency as all the others. The outputs of all the cavities are combined and brought in phase to the front end of a single low-noise cryogenic amplifier. Isolators may be used to avoid coupling among cavities and subsequent mode splitting. The price one has to pay for failing to keep all the cavities at exactly the same frequency is a broadening of the bandwidth in which one is searching for a signal, and hence an increase in noise. This corresponds to a decrease in the effective cavity quality factor Q_ω that appears in Eq. (4). Provided there is no degradation in Q_ω due to a lack of perfect tuning between all the cavities, the search rate for a given signal-to-noise [Eq. (4)] changes with frequency as in method 4, i.e., $V \sim f^0$, $c \sim f^0$, $Q_\omega \sim f^{-2/3}$ and hence

$$\frac{d\ln f}{df} \sim f^{1/3} T_n^{-2}(f). \tag{20}$$

This method has no problem with transverse mode localization. At sufficiently high frequencies, it will however encounter the problems of longitudinal mode localization and resonance crossing. Of the two, the former appears the most severe. Its expression is Eq. (9). The problem of longitudinal localization (as well as the problem of resonance crossing) can in turn be solved by dividing the cavities longitudinally as well as transversely.

III. DISCUSSION

We have discussed design considerations for the cavities which would be used in a detector for galactic halo axions. Such cavities must be tunable over rather large frequency ranges (say 20% tuning range per cavity) and must be such that the quantity C^2Q is as large as possible at all frequencies. Mode localization is the tendency of a cavity mode to localize in a small region of the cavity unless special precautions are taken to avoid this. Mode localization reduces the effective volume of the cavity and hence degrades C^2Q. In particular, to avoid longitudinal mode localization one must maintain at all times a minimum standard of longitudinal uniformity. Equation (9) expresses the corresponding restraint. Tuning mechanisms should also maintain the longitudinal translational symmetry of the cavity, implying that it is better to translate a dielectric tuning rod sideways along the transverse direction than to insert it along the axis of the cavity. One can also tune by translating sideways a longitudinal metal post of full cavity length placed inside the cavity. In fact this method yields larger tuning ranges and larger C^2Q values than the translating of a dielectric rod.

The best method to extend the search to high frequency may be the brute-force approach of ganging together many small cavities all resonant at the same frequency. This method does entirely avoid the problems of mode localization and resonance crowding, although it does so only at the cost of rapidly increasing complexity. Alternatively, one can build "multi-cell" cavities, e.g., cavities in which longitudinal metallic posts are judiciously placed. Using computer simulations, we have optimized the radii and positions of these posts to obtain the largest possible C^2Q values. The results are given in Fig. 13. However, at frequencies larger than two or three times the frequency of the fundamental TM mode of the cavity without metal posts, it becomes increasingly difficult to avoid transverse mode localization associated with slight deviations of the radii and positions of the metal posts from their optimum values. At those frequencies, the problem of mode crossings becomes increasingly severe as well.

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