PHY 4324—-Electromagnetism 2–Fall 2019

Problem set 1—Due Friday, September 6th

10 points each problem. Parts with * are more strongly weighted.

1. Let's take Eq. 7.6 as a good model for electrical conduction in copper metal. The term in parentheses is the conductivity, as in Ohm's law $\mathbf{J} = \sigma\mathbf{E}$. Now, copper has resistivity $\rho = 1/\sigma = 1.68 \times 10^{-8}$ Ω-m and has $nf = 8.5 \times 10^{28}$ electrons per cubic meter. (Only one electron per copper atom is free to move.) The charge q and mass m are those of the electron.

   a. Substitute $\tau = \lambda/v$ in the equation for the conductivity. τ is the time between collisions with λ the distance the electron travels between collisions when moving at velocity v (or v_{thermal}). Find an equation for τ in terms of the quantities above.

   b. Find a numerical value for τ in sec.

   c. Use the equipartition theorem of classical statistical mechanics, $\frac{3}{2}kT = \frac{1}{2}mv^2$ to estimate the thermal velocity at $T = 300$ K. Here k is Boltzmann's constant.

   d. From this and τ, find λ.

   e. Instead of this thermal velocity, use the quantum mechanical Fermi velocity, which is $c/200$, with c the speed of light, to find λ.

   f. Compare the mean free path to the spacing between atoms. You can either look up the lattice constant of copper or you can say that if you know n, the number of atoms in a unit volume, then you know the volume associated with a single atom. Then think about the atoms as being in the center of little cubes stacked together and use this idea to arrive at spacing.

2. Because like charges repel each other, clumps of charge in a conductor will spread out to a uniform background. Here you will calculated how rapidly this happens. Consider a material that obeys Ohm's law and has a conductivity σ and permittivity ϵ. Start with an amount of free charge within a small volume within this material. (This volume might as well be at the origin, at $\mathbf{r} = 0$.) There is a free charge density ρ_0 in this volume at time $t = 0$.

   a. What is the charge density there (at $\mathbf{r} = 0$) at later times?

   b. Define a time constant τ for the decay of the charge density.

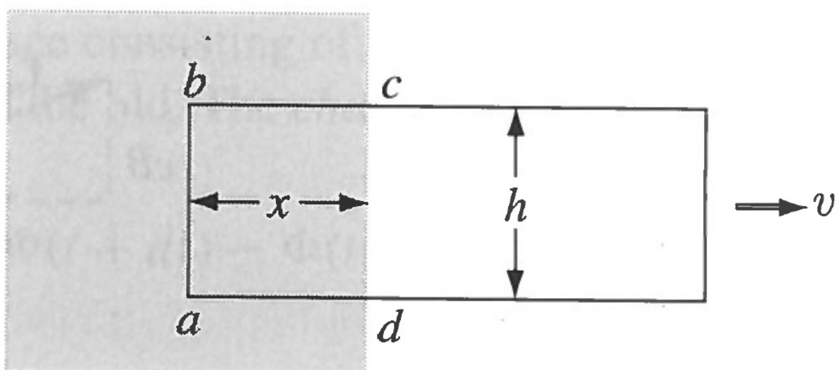
   c. Calculate the time constant for the case of a good metal like copper ($\rho = 1/\sigma = 1.68 \times 10^{-8}$ Ω-m and $\epsilon_r = 1$.)

   d. Calculate the time constant for the case of a semiconductor like silicon ($\rho = 1/\sigma = 2500$ Ω-m and $\epsilon_r = 11.7$.)

   e. Calculate the time constant for the case of a good insulator like Teflon ($\rho = 1/\sigma = 1. \times 10^{22}$ Ω-m and $\epsilon_r = 3$.)

   Hint: Use the continuity equation and Gauss' law. Recognize that ρ is used for both charge density and resistivity.

3. A superconductor is a metal with zero resistance. It's a quantum phenomenon, beyond the scope of our class. But we can consider a *perfect conductor.* The perfect conductor has zero resistance, so $\rho = 0$ and $\sigma = \infty$.
   a. In electrostatics, the electric field is zero inside a conductor. Show that the electric field is zero inside the perfect conductor, even if a steady (dc) current is flowing. (Hint: this is trivial; the current is finite.)
   b. Suppose there is a magnetic field inside the perfect conductor. Show that this magnetic field cannot change, i.e., $\frac{\partial B}{\partial t} = 0$.
   c. Now make a loop from perfectly conducting wire. Show that the magnetic flux through this loop is constant.
4. Consider the motion of a loop in a magnetic field $\mathbf{B}$, shown below. The loop is moving to the right with velocity v. Suppose that the loop is made from perfectly conducting wire, so that $R = 0$. The loop has height h, inductance L, and mass m. The current in the loop is not infinite; it is limited by the back emf of the loop's self inductance.
   a. Show that the loop undergoes simple harmonic motion.
   b. Find the frequency of this motion in terms of $\mathbf{B}$, h, L, and m.



This is Fig. 7.10 from the text, but with $R = 0$. The magnetic field $\mathbf{B}$ is into the page.