Problem set 1—Due Wednesday, January 25 at 11:59 pm on Canvas. (See syllabus for the policy on homework turned in late.) 10 points each problem.

For each problem where a numerical value is requested, first work out an equation for the requested quantity and second put in numerical values for known quantities to get a numerical answer. Both are required.

1. A factor that appears in many of the formulas of relativity is the “Lorentz term,” usually called $\gamma$,

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In relativity calculations, one often has to deal with two numbers that have very large relative (!) values. For example $c = 2.9979 \times 10^8$ m/s whereas an ant might move at $v = 1.0000 \times 10^{-3}$ m/s (1 mm/s). Find $\gamma - 1$ and $\gamma^4 - 1$ for the ant, accurate to 3 significant figures. (Give simplified equations and then the numbers. Justify the simplifications you used.)

[*Our textbook does not define or use $\gamma$ but you see $\sqrt{1 - v^2/c^2}$ in the denominator of many equations in Chapter 2. Krane is in the minority; almost all discussions of special relativity introduce $\gamma$ and some use it without definition, assuming “everyone knows that.” Another function commonly seen is $\beta \equiv v/c$. A useful feature of $\gamma$ and $\beta$ is that they are pure numbers, independent of the units used for speed.*]

2. An observer, called C, sees two spaceships, A and B, traveling along the observer’s x axis with relative speeds of 0.6c and 0.9c.

   a. What is the speed of one spaceship relative to the other spaceship if they are going in the same direction, say along $+x$?
   
   b. What is their relative speed if they are going in opposite directions?
   
   c. A stick of length $\ell_0$ is carried by A. What is the length of the stick according to C?
   
   d. What is the length of the stick according to B if A and B are going in the same direction, say along $+x$?
   
   e. What is the length of the stick according to B if A and B are going in opposite directions?

3. The light emitted by stars is governed by (at least) two kinds of processes. One is a continuum of “black body” radiation, with a spectrum determined by the temperature of the star. (Our local star, the Sun, has $T \approx 6000$ K and appears yellow.) Atoms in the atmosphere of stars absorb light at specific wavelengths, according to the quantum mechanics of the atom. (We will get to these two ideas shortly. Google “light from the sun” and it will find you a spectrum.)

   Astronomers on Earth observe light emitted by a star some distance away. They determine that these atomic absorption features appear at wavelengths that are four (4.0) times those of the features from our Sun. (Ignore the Earth’s orbital and rotational motion in working this problem.)
a. What is the speed at which the star is receding from the Earth and the Sun?
b. From the point of view of beings on a planet orbiting this star, what is the speed at which our Sun recedes from them? Explain your answer.

4. Cosmic rays entering the atmosphere generate muons, $\mu^\pm$. Actually, they generate lots of stuff, as show below, but the interest here is in the muons. (If you take PHY 4803L you have a chance to measure the muon flux in one of the experiments there.)

![Cosmic ray shower](image)

The muon, a sort of heavy electron, is unstable. In its rest frame, a muon has a lifetime of

$$t_{1/2} = 2.2 \mu\text{sec}.$$  

Let’s assume (1) that a muon is created 3 km above the Earth’s surface, (2) that it is traveling straight down, (3) that its speed is $v_\mu = 0.98c$, and (4) that it decays in its own frame exactly 2.2 $\mu$sec after creation.

a. Classically, how far would the muon travel in $t_{1/2}$? (Equation and number.)
b. Classically, how long would the muon need to survive to reach the surface?
c. Now work the problem with relativity. What is the factor $\gamma$ for this muon? (See problem 1 for the definition of $\gamma$.)
d. According to an observer on Earth, how long will the muon survive before decaying?
e. Will the muon reach the Earth’s surface?
f. According to the muon itself (or an observer riding in the frame of the muon) what is the distance above the Earth at which the muon was created?