PHY 4324—-Electromagnetism 2–Fall 2019

Problem set 2—Due Friday, September 20th

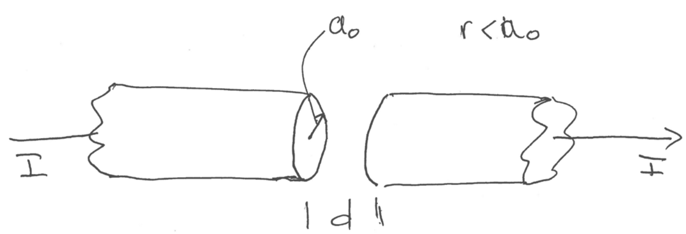
10 points each problem.

1. A long wire, with finite radius a_0, has, somewhere along the wire, a break, leading to a gap of thickness d in the wire. You may think of the break as a parallel-plate capacitor with plate area πa_0^2 and gap d. A constant current I flows in the wire, starting at time $t = 0$, charging the capacitor. The source of current in the wire is an ideal current source, so that I is constant, even as the capacitor charges up.

   Hint: Ignore fringing fields.

   Hint: This is a case where it may be profitable to think of the displacement current as an actual current.

   Hint: All the calculations call for finding things for radius $r < a_0$.

   a. Find the magnetic field in the gap at a distance from the axis r where $r < a_0$.
   b. Find the electric field in the gap as a function of time.
   c. Find the energy density in the gap.
   d. Calculate the Poynting vector in the gap. (Magnitude and direction.)
   e. Show that the continuity equation for energy is satisfied.
   f. Calculate the total energy in the gap as a function of time. Also calculate the rate of change of energy, $\frac{dU}{dt}$.
   g. Find the power flowing into the gap from an integral of the Poynting vector and compare with $\frac{dU}{dt}$.

2. Let's start with a parallel-plate capacitor in vacuum, having capacitance C_0 and charged to voltage V_0. The plates are separated by distance t_0 and have area A. (So now you know the charge, surface charge density, and electric field vector between the plates.) A magnetic field $\mathbf{B_0}$ is *parallel to the surface of the plates.*

   a. What is the electromagnetic momentum in the volume between the plates?
   b. Now a conducting wire is connected between the plates. The wire has resistivity ρ and radius a_0. (Of course its length is t_0.) The capacitor discharges. From circuit theory you know that the discharge current is exponential in time, with time constant

$\tau = RC_0$ witb R the wire resistance. Because of the magnetic field, there will be a force on the wire during the discharge. What is the tim-dependent force on the wire?

c. Assume that the capacitor is in otherwise empty space, so that it is not fixed to any support. It is at rest when the wire is connected. What is its momentum at the end of the discharge?

3. Two point charges, both with charge $+q_0$ are fixed on the x axis, one at $x = +a_0$ and the other at $x = -a_0$.

   a. Integrate the Maxwell stress tensor over the y-z plane to find the force of one charge on the other. (The forces are equal and opposite, so you only need to do one.)

   b. Now let the charge at $x = -a_0$ have value $-q_0$. Do the same calculation.

4. Consider the following functions $f(x, t)$ and show whether they do or do not satisfy the wave equation in one dimension. (x is the coordinate along the single space axis. t is the time. v is a velocity. k is a constant with dimensions of [m^{-1}] there to make the arguments dimensionless.

   a. $f = e^{(kx-kvt)^2}$

   b. $f = \sin(kx - kvt)$

   c. $f = \cos(kx - kvt)$

   d. $f = \tan(kx - kvt)$

   e. $f = [(kx - kvt)^2 + 1]^{-1}$

   f. $f = \sin(kx)\cos(kvt)$

   g. $f = \sin(kx)\cos^2(kvt)$

   h. $f = e^{-(k^2x^2+k^2v^2t^2)}$

   Hint: Try each function in the wave equation. If it is satisfied, so you get $0 = 0$, you can say "yes." Otherwise, "no."