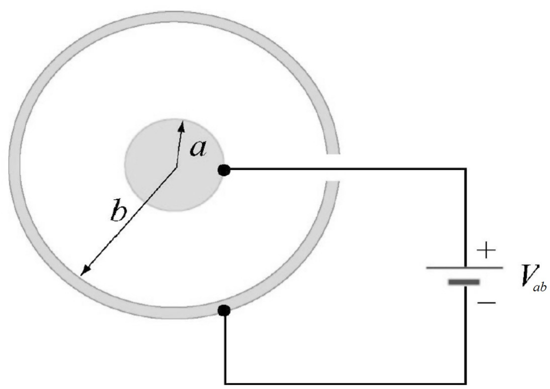
PHY 4324—-Electromagnetism 2–Spring 2021

Problem set 1—Due Friday, January 22nd at 10:00 pm

10 points each problem.

You must scan your solutions and upload to Canvas by the due time. I will upload solutions shortly afterward and homework turned late will not be accepted.

1. Start by reviewing Example 2.12 in Griffiths, where the capacitance between two concentric spherical metal shells is calculated. Now, let us take this capacitor and fill the space between the two shells with a conducting material (conductivity σ, not too big). The dielectric constant of this material is $\epsilon = \epsilon_0$. The conductivity of the metal of the shells is very large. The device is shown in the image below.



The inner sphere has radius a while the inside radius of the outer sphere is b. An external circuit maintains a potential difference V_{ab} between the spheres. (Ignore the fact that a small hole is needed to connect the inner sphere to the circuit.)

a. Assume that the shells have total charge Q on their surfaces. One is positive and one negative. Which is which?
b. Use your knowledge of electrostatics to find the electric field and electrostatic potential at a radius r between a and b in terms of Q.
c. Find the value of Q in terms of V_{ab}.
d. Find the current density $\vec{j}$ at r in terms of σ and V_{ab}.
e. Find the total current flowing between the spheres.
f. What is the resistance between the shells?
g. If $b \gg a$ show that the outer radius (b) is irrelevant. What is the resistance in this case?
h. Look up the capacitance in Chap. 2 and calculate the product RC in terms of σ and ϵ_0. Check the units. RC is a time.

. For your own thinking about the physics, read Problem 7.3. It seems that the product RC is independent of the shape, size, or orientation of the two metallic objects.

2. We have found Ohm's law to be $\vec{j} = \sigma \vec{E}$ where $\vec{j}$ is the current density, $\vec{E}$ the electric field, and σ the conductivity of the current-carrying material. Suppose $\vec{j}$ is constant but σ increases. (You can make this happen by lowering temperature of the conductor towards absolute zero.) Then $\vec{E}$ must be smaller.

   a. Take the limit as $\sigma \to \infty$, making the material a perfect conductor, and find the value of $\vec{E}$ in the material. (Remember $\vec{j}$ is constant.)

   b. Show that the magnetic field inside a perfect conductor can never change.

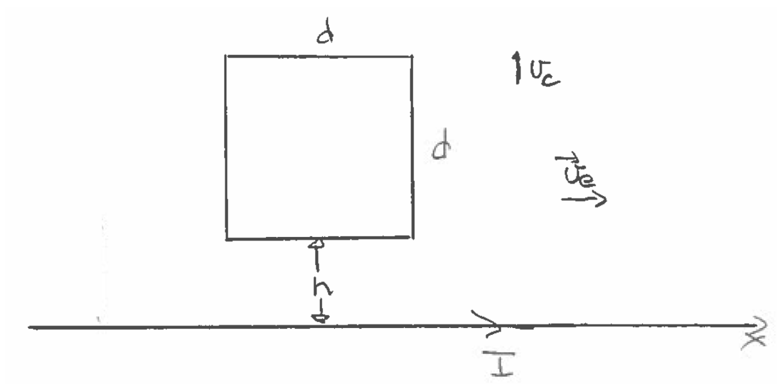
   c. Now lets make a loop of the perfect conductor in a magnetic field. Show that after the loop is closed, the magnetic flux enclosed by the loop is constant.

3. Consider a square loop in a *non-uniform* and *time varying* field. The field is given by

$$\vec{B} = Cty^4 \hat{z}$$

   where y is measured from the bottom of the loop and C is a constant. The loop is in the x-y plane. and has edge dimension d. Calculate the emf $\mathcal{E}$ induced in the loop.

4. A long (infinitely long) straight wire is oriented along the x axis. The wire carries current I. A square loop, with edge dimension d is located a distance h above the loop. See the diagram below.



   a. Write down (from last semester) the magnetic field as a function of distance s from the wire.

   b. Calculate the total magnetic flux Φ through the loop.

   c. Now, the loop is pulled at velocity $\vec{v}_c$ directly away from the wire. Calculate the emf in the loop.

   d. What is the direction of current flow (clockwise or counterclockwise in the picture)?

   e. Suppose the loop is pulled at velocity $\vec{v}_e$ in a direction parallel to the wire (left or right, doesn't matter). What is the induced emf?