PHY 4324—-Electromagnetism 2–Spring 2021

Problem set 2—Due Wednesday, February 3rd at 10:00 pm

10 points each problem.

You must scan your solutions and upload to Canvas by the due time. I will upload solutions shortly afterward and homework turned late will not be accepted.

1. A long solenoid magnet has radius r, length D. It has N turns. The current in the wire of the solenoid is I. Find the energy stored in this solenoid in 4 different ways.
   a. From the inductance and current, Eq. 7.30.
   b. From the vector potential and current, Eq. 7.31. (See example 5.12 for the vector potential.)
   c. From the volume integral of B^2 and a surface integral of $A \times B$, Eq. 7.34. (Take the volume as a hollow cylinder with inner radius $a < r$ and an outer radius $b > r$.)
   d. From the volume integral (all space) of B^2, Eq. 7.35.
2. Let's think about using water to increase the capacitance of a parallel plate capacitor. The dielectric constant of water is $\epsilon = 81\epsilon_0$, so the increase should be significant. The person doing this experiment was careless, and the water had a concentration of ions in solution, making it conductive, with a conductivity of $\sigma = 4.0\ \Omega^{-1}\mathrm{m}^{-1}$. Take the permeability to be $\mu = \mu_0$. The capacitor is attached to a voltage source with $V = V_0 \cos(2\pi f t)$ with $V_0 = 5$ V. The capacitor has plate area of 0.1 m^2 and plate spacing of 0.01 m.
   a. Derive equations for the conduction current density and displacement current density.
   b. Calculate the conduction current and displacement current when $f = 860$ kHz (WRUF-AM).
   c. Calculate the conduction current and displacement current when $f = 2$ GHz (a common cell phone frequency).
   d. Calculate the conduction current and displacement current when $f = 20$ THz (an infrared light frequency).
3. Let's start with a parallel-plate capacitor in vacuum, having capacitance C_0 and charged to voltage V_0. The plates are separated by distance t_0 and have area A. (So now you know the charge, surface charge density, and electric field vector between the plates.) A magnetic field $\mathbf{B_0}$ is *parallel to the surface of the plates.*
   a. What is the electromagnetic momentum in the volume between the plates?
   b. Now a conducting wire is connected between the plates. The wire has resistivity ρ and radius a_0. (Of course its length is t_0.) The capacitor discharges. From circuit theory you know that the discharge current is exponential in time, with time constant $\tau = RC_0$ witb R the wire resistance. Because of the magnetic field, there will be a force on the wire during the discharge. What is the time-dependent force on the wire?

c. Assume that the capacitor is in otherwise empty space, so that it is not fixed to any support. It is at rest when the wire is connected. What is its momentum at the end of the discharge?

4. Two point charges, both with charge $+q_0$ are fixed on the x axis, one at $x = +a_0$ and the other at $x = -a_0$.

   a. Integrate the Maxwell stress tensor over the y-z plane to find the force of one charge on the other. (The forces are equal and opposite, so you only need to do one.)

   b. Now let the charge at $x = -a_0$ have value $-q_0$. Do the same calculation.