PHY 4324—-Electromagnetism 2–Spring 2021

Problem set 4—Due Monday, March 1st at 10:00 pm

10 points each problem.

You must scan your solutions and upload to Canvas by the due time. I will upload solutions shortly afterward and homework turned late will not be accepted.

1. We showed that plane electromagnetic waves are solutions to Maxwell's equations. Here, consider a *spherical wave*, :

$$\vec{E}(\vec{r},t) = A\frac{\sin\theta}{r}\left[\cos(kr-\omega t) - \frac{\sin(kr-\omega t)}{kr}\right]\hat{\phi}$$

traveling in vacuum, where $\omega = kc$. The wave is written in spherical coordinates.

   a. What is the direction of the vector $\vec{k}$? (Hint, a general wave in three-dimensions is of the form $f(\vec{k}\cdot\vec{r}-\omega t)$.)

   b. Show that $\nabla\cdot\vec{E} = 0$. (Hint: we cannot use the equations for plane waves to substitute $\nabla \to i\vec{k}$.)

   c. Find $\vec{B}$ by using Faraday's law (and by integrating over time). Calculate $\hat{k}\times\hat{e}$. It should be in the direction of $\hat{b}$.

   d. Show that $\nabla\cdot\vec{B} = 0$.

   e. Substitute $\vec{E}$ and $\vec{B}$ into Ampere's law and show that it is satisfied.

   f. Calculate the Poynting vector $\vec{S}$.

   g. Average $\vec{S}$ over a cycle (t from zero to $2\pi/\omega$) to find $\langle\vec{S}\rangle$.

   h. What is the direction of the time average Poynting vector $\langle\vec{S}\rangle$? How does it depend on r?

   i. Compute the integral of $\langle S\rangle\cdot\hat{r}$ over a spherical surface of radius r. ($\hat{r}$ is the normal to that surface.) Does it depend on the value of r?

2. Consider a interface between vacuum and a medium with $\epsilon \neq \epsilon_0$ and $\mu \neq \mu_0$. (It has both permittivity and permeability.) Both ϵ and μ are real.

   a. Calculate the normal-incidence amplitude reflectivity (ratio of electric fields) of the interface.

   b. Calculate the reflectance $\mathscr{R}$ (ratio of intensities).

   c. Calculate the normal-incidence amplitude transmission coefficient (ratio of electric fields).

   d. Calculate the transmittance $\mathscr{T}$ (ratio of intensities).

   e. Confirm that $\mathscr{R}+\mathscr{T} = 1$.

3. There are a number of science fiction stories about “slow glass.” Written by Bob Shaw, they describe slow glass as having a huge refractive index, so that light takes, say, 1 year to travel through a centimeter of the glass. You could set a pane of slow glass overlooking a beautiful scene, and a year later the light which struck the front surface would emerge from the back. You could then take the glass and install it in your house and, for a year, enjoy the view while far from the scene. Here are some questions about slow glass.

   a. Calculate the refractive index of slow glass. Keep 5 significant digits.

   b. What would the normal-incidence reflectance $\mathscr{R}$ of the glass surface be if there is no antireflection coating applied to reduce it? Hint: Many calculators do not have enough precision to calculate $\mathscr{R}$ from the equation in the book. It may be easier to write $1 - \mathscr{R}$ as a ratio of two quantities, calculate its numerical value, and then compute $\mathscr{R}$.

   c. For slow glass to be usable there would have to be a coating on both sides of the glass to reduce the reflectance to zero. Assume such a coating. Suppose a square meter of the 1-cm thick glass is set in space where the Sun shines on it 24/7. After a year of exposure, how much energy would be stored in the glass? (The Sun delivers about 1 kW/m^2 to the Earth.)

   d. Estimate the electric and magnetic field strengths inside the glass from the stored energy density. You can use $B = E/v$ to get the partition into electric and magnetic energies. Take $\mu = \mu_0$.

4. This problem asks four questions about electromagnetic properties of glass, silicon, and silver. You’ll need to know the relative dielectric constant (or the refractive index) and the electrical resistivity (or the conductivity). You can find these in Griffiths, except for the relative dielectric constant of silver. Take that to be $\epsilon_R = 1.0$. Take $\mu = \mu_0$ for all.

   a. The conductivity of silver is higher than any other metal. Consequently it is used in microwave cables and waveguides to minimize loss. Suppose your microwave circuit is operating at 30 GHz. You want to use silver but no more than necessary because of the cost. How thick would you make the silver coatings?

   b. Now reduce the frequency to 1 MHz. What is the wavelength of electromagnetic waves in silver at this frequency? What is the speed of the waves in the silver? Compare these to waves in vacuum.

   c. Glass has a high resistivity, with a range of values. Suppose a block of glass has some free charge embedded in it. How long would it take for the charge to diffuse to the surface. (Use the glass resistivity value that gives the shortest time.)

   d. Silicon is quite transparent in the infrared, at wavelengths longer than about 1.1 μm. calculate for silicon in the infrared (1) the normal-incidence amplitude reflection coefficient (ratio of fields), (2) the normal-incidence intensity reflection coefficient (the reflectance), (3) Brewster’s angle, and (4) the critical angle for total internal reflection.