Asymptotic Freedom in Yang-Mills from Open String Loops

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Charles Thorn

IAS, University of Florida

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Introduction: Field/String Duality

Much to learn from close relationship of string and field theory.

AdS/CFT asserts equivalence, with QFT the simpler description for $Ng^2 \ll 1$, and string description simpler for $Ng^2 \gg 1$

My work with Bardakci attacks $Ng^2 = O(1)$ by mapping planar diagrams of Yang-Mills to a worldsheet template.

Summing planar open string multi-loop diagrms no harder in this framework, but conceptually more promising

Today's talk devoted to exploring the field theory limit of the open string planar 1-loop diagrams.

Brief Review of Field/String Duality

’t Hooft’s $N \to \infty$ (SU(N) Chan-Paton):

$$\sum (\text{Planar Open String Loops})_{D3} \equiv \sum (\text{Closed String Trees})_{D3\text{source}}$$

Left Side $\underset{\alpha' \to 0}{\Longrightarrow}$ $N = \infty$ Gauge Theory in 4d

Right Side $\underset{\substack{\alpha' \to 0 \\ g^2 N \to \infty}}{\Longrightarrow}$ Classical gravity

If $g^2 N = O(1)$, right side stays stringy as $\alpha' \to 0$.
I.e. must solve classical closed string field eqs.

My suggestion:

$N \to \infty$ QCD by direct planar graph summation at $\alpha' > 0$.

Simplest Open String for Pure 4D Yang-Mills

Even G-parity sector of Neveu-Schwarz open string in $D = 4$

No fermion R sector, no extra dimensions

Gauge group: SU(N) Chan-Paton factors

Rest of talk: One-loop Yang-Mills from NS+ Model

Method of Metsaev&Tseytlin applied to bosonic string:
Nucl Phys B**298** (1988) 109.

The following 23 slides include a **complete** calculation of asymptotic freedom: Don't try to follow every detail! I will make them available.

Some Notation

$$k^{\pm} = \frac{k^0 \pm k^3}{\sqrt{2}}, \qquad k^{\wedge(\vee)} = \frac{k^1 + (-)ik^2}{\sqrt{2}}$$

$$K_{lm}^{\mu} \equiv k_l^+ k_m^{\mu} - k_m^+ k_l^{\mu} \qquad \text{for each pair } (l, m)$$

Let ϵ_m^{μ} be a gluon polarization with $\epsilon_m^+ = 0$.

Then $\epsilon_m^- = (k_m^{\wedge}\epsilon^{\vee} + k_m^{\vee}\epsilon^{\wedge})/k_m^+$

$$k_l \cdot \epsilon_m = \frac{K_{ml}^{\wedge}\epsilon_m^{\vee} + K_{ml}^{\vee}\epsilon_m^{\wedge}}{k_m^+}$$

Consider 3 lightlike momenta with $\sum_{i=1}^{3} k_i^{\mu} = 0$

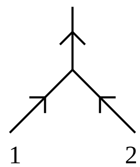
for $\mu = \wedge, \vee, +$. Then $K_{12}^{\wedge,\vee} = K_{23}^{\wedge,\vee} = K_{31}^{\wedge,\vee}$, and

$$\sum_{i=1}^{3} k_i^- = \frac{K_{12}^{\wedge} K_{12}^{\vee}}{k_1^+ k_2^+ (k_1^+ + k_2^+)}$$

Conservation $\Leftrightarrow K_{12}^{\wedge} = 0$

Gluon Vertex Operator in NS+ Model

$$V(k,\epsilon) = g\sqrt{\frac{2}{\alpha'}}(\epsilon\cdot\mathcal{P} + \sqrt{2\alpha'}k\cdot H\epsilon\cdot H)e^{ik\cdot x}$$

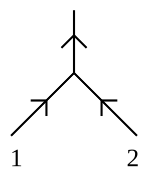


$$= \langle 0,k_1|\epsilon_1\cdot b_{1/2}V(k_2,\epsilon_2)\epsilon_3\cdot b_{-1/2}|0,k_3\rangle$$

$$= 2g(\epsilon_1\cdot\epsilon_3 k_3\cdot\epsilon_2 + k_2\cdot\epsilon_1\epsilon_2\cdot\epsilon_3 - k_2\cdot\epsilon_3\epsilon_1\cdot\epsilon_2)$$

Choose Lightcone gauge $\epsilon_i^+ = 0$; then

$$\begin{aligned} \epsilon_1^\wedge &= \epsilon_2^\wedge = \epsilon_3^\vee = 0, \quad \epsilon_1^\vee = \epsilon_2^\vee = \epsilon_3^\wedge = 1 \\ \epsilon_1 \cdot \epsilon_2 &= 0, \quad \epsilon_1 \cdot \epsilon_3 = \epsilon_2 \cdot \epsilon_3 = 1 \end{aligned}$$



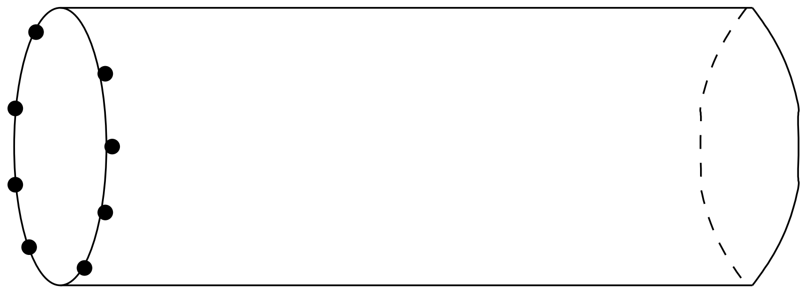
$$= \quad 2g \frac{k_1^+ + k_2^+}{k_1^+ k_2^+} K_{12}^\wedge$$

Taking this vertex in planar graphs ($N_c \to \infty$) shows that

$$\alpha_s N_c \equiv \frac{g_s^2 N_c}{4\pi} = \frac{g^2}{2\pi}$$

g_s is conventional QCD coupling,
N_c is number of colors.

NS+ at 1 loop, $D < 10$ (Goddard and Waltz, 1971)



One loop planar M gluon NS+ amplitude for $D < 10$:

$$\mathcal{A}_{1\ \text{loop}} = (g\sqrt{2\alpha'})^M \frac{\mathcal{M}^+ - \mathcal{M}^-}{2}$$

where, in cylinder variables, $\ln q = 2\pi^2/\ln w$,

$$
\begin{aligned}
\mathcal{M}^{+} &= 2\left(\frac{1}{8\pi^2\alpha'}\right)^{D/2} \int \prod_{k=2}^{M} d\theta_k \int_0^1 \frac{dq}{q} \\
&\quad \sqrt{\frac{-\pi}{\ln q}} q^{-(D-1)/8} w^{(D-9)/16} (1 - w^{1/2}) \\
&\quad \frac{\prod_r (1+q^{2r})^{D-1}}{\prod_n (1-q^{2n})^{D-1}} \prod_{l<m} \left[\psi(\theta_m - \theta_l, q)\right]^{2\alpha' k_l \cdot k_m} \\
&\quad \langle \hat{\mathcal{P}}_1 \hat{\mathcal{P}}_2 \cdots \hat{\mathcal{P}}_M \rangle^{+}
\end{aligned}
$$

$q^{-(D-1)/8}$ reflects closed string tachyon: $\alpha' p^2 = (D-1)/4$

Graviton is massive: $\alpha' p^2 = (D-1)/4 - 2 = (D-9)/4$

$$
\begin{aligned}
\mathcal{M}^- &= 2\left(\frac{1}{8\pi^2\alpha'}\right)^{D/2}\int\prod_{k=2}^{M}d\theta_k\int_0^1\frac{dq}{q} \\
&\quad \sqrt{\frac{-\pi}{\ln q}}2^{(D-1)/2}w^{(D-9)/16}(1+w^{1/2}) \\
&\quad \frac{\prod_n(1+q^{2n})^{D-1}}{\prod_n(1-q^{2n})^{D-1}}\prod_{l<m}\left[\psi(\theta_m-\theta_l,q)\right]^{2\alpha' k_l\cdot k_m} \\
&\quad \langle\hat{\mathcal{P}}_1\hat{\mathcal{P}}_2\cdots\hat{\mathcal{P}}_M\rangle^-
\end{aligned}
$$

where

$$
\begin{aligned}
\psi(\theta,q) &= \sin\frac{\theta}{2}\prod_n\frac{(1-q^{2n}e^{i\theta})(1-q^{2n}e^{-i\theta})}{(1-q^{2n})^2} \\
\hat{\mathcal{P}} &= \epsilon\cdot\mathcal{P}+\sqrt{2\alpha'}k\cdot H\epsilon\cdot H
\end{aligned}
$$

Range of integration: $0=\theta_1<\theta_2<\cdots<\theta_N<2\pi$

$\langle \cdots \rangle$ evaluated via Wick expansion with contractions:

$$\begin{aligned}
\langle \mathcal{P}_l \rangle &= \sqrt{2\alpha'} \sum_i k_i \left[\frac{1}{2} \cot \frac{\theta_{il}}{2} + \sum_{n=1}^{\infty} \frac{2q^{2n}}{1-q^{2n}} \sin n\theta_{il} \right] \\
\langle \mathcal{P}_i \mathcal{P}_l \rangle &= \langle \mathcal{P}_i \rangle \langle \mathcal{P}_l \rangle + \frac{1}{4} \csc^2 \frac{\theta_{il}}{2} - \sum_{n=1}^{\infty} n \frac{2q^{2n}}{1-q^{2n}} \cos n\theta_{il} \\
\langle H_i H_j \rangle^+ &= \frac{1}{2\sin(\theta_{ji}/2)} - 2 \sum_r \frac{q^{2r} \sin r\theta_{ji}}{1+q^{2r}} \\
\langle H_i H_j \rangle^- &= \frac{\cos(\theta_{ji}/2)}{2\sin(\theta_{ji}/2)} - 2 \sum_n \frac{q^{2n} \sin n\theta_{ji}}{1+q^{2n}}
\end{aligned}$$

Space-time indices are suppressed

In these formulas r ranges over positive half odd integers, n over positive integers, and $l, m \in [1, \cdots, M]$.

The UV Cutoff

Metsaev and Tseytlin: $-\ln w$ is like a Schwinger parameter T:

$$\frac{\alpha'}{L_0 - 1/2} = \int_0^\infty dT e^{-(L_0-1/2)T/\alpha'}$$

UV cutoff:

$$T > \frac{1}{\Lambda^2} \;\Leftrightarrow\; -\ln w > \frac{1}{\alpha'\Lambda^2} \;\Leftrightarrow\; -\ln q < 2\pi^2\alpha'\Lambda^2$$

Field Theory limit is $\alpha' \to 0$ at fixed Λ.

1 Loop 2 Gluon Amplitude

$$\langle \hat{\mathcal{P}}_1 \hat{\mathcal{P}}_2 \rangle = \langle \epsilon_1 \cdot \mathcal{P}_1 \epsilon_2 \cdot \mathcal{P}_2 \rangle + 2\alpha' \langle k_1 \cdot H_1 \epsilon_1 \cdot H_1 k_2 \cdot H_2 \epsilon_2 \cdot H_2 \rangle$$

$$\begin{aligned} \langle \epsilon_1 \cdot \mathcal{P}_1 \epsilon_2 \cdot \mathcal{P}_2 \rangle &= \epsilon_1 \cdot \epsilon_2 \left[\frac{1}{4} \csc^2 \frac{\theta}{2} - \sum_{n=1}^{\infty} \frac{2nq^{2n} \cos n\theta}{1-q^{2n}} \right] \\ &\quad -2\alpha' k_2 \cdot \epsilon_1 k_1 \cdot \epsilon_2 \left[\frac{1}{2} \cot \frac{\theta}{2} + \sum_{n=1}^{\infty} \frac{2q^{2n} \sin n\theta}{1-q^{2n}} \right]^2 \end{aligned}$$

$$\langle k_1 \cdot H_1 \epsilon_1 \cdot H_1 k_2 \cdot H_2 \epsilon_2 \cdot H_2 \rangle = (k_2 \cdot \epsilon_1 k_1 \cdot \epsilon_2 - k_1 \cdot k_2 \epsilon_1 \cdot \epsilon_2) C^{\pm}$$

$$\begin{aligned} C^+ &= \left[\frac{1}{2\sin(\theta/2)} - 2\sum_r \frac{q^{2r} \sin r\theta}{1+q^{2r}} \right]^2 \\ C^- &= \left[\frac{\cos(\theta/2)}{2\sin(\theta/2)} - 2\sum_n \frac{q^{2n} \sin n\theta}{1+q^{2n}} \right]^2 \end{aligned}$$

Divergences in θ Integral

Momentum conservation: $k_2 = -k_1$, $k_1 \cdot k_2 = k_i \cdot \epsilon_j = 0$
θ integral reduces to:

$$\epsilon_1 \cdot \epsilon_2 \int_0^{2\pi} d\theta \left[\frac{1}{4} \csc^2 \frac{\theta}{2} - \sum_{n=1}^{\infty} \frac{2nq^{2n} \cos n\theta}{1 - q^{2n}} \right] = \infty$$

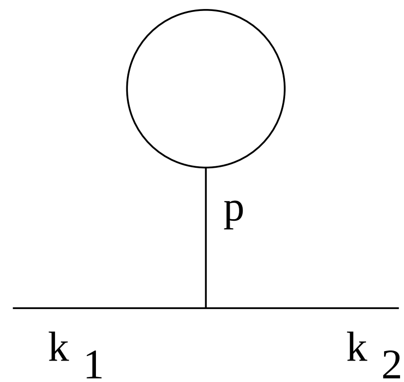
Goddard and Neveu/Scherk suggest calculating with $k_1 + k_2 = p \neq 0$, and $p \to 0$ at the end. (GNS regularization)

$$\int_0^{2\pi} \frac{d\theta}{4} \csc^2 \frac{\theta}{2} \psi(q,\theta)^{2\alpha' k_1 \cdot k_2} = \int_0^{\pi} \frac{d\theta}{2} (\sin\theta)^{\alpha' p^2 - 2} + O(p^2)$$
$$= \frac{\Gamma(1/2)\Gamma(-1/2 + \alpha' p^2/2)}{2\Gamma(\alpha' p^2/2)} + O(p^2)$$
$$\sim -\pi\alpha' p^2/2 + O(p^2)$$

As first noted by Neveu and Scherk for the bosonic string (1972)

Poles in p^2

$\Gamma(-1/2 + \alpha' p^2/2)$ has poles: $\alpha' p^2 = 1 - 2n$, $n = 0, 1, 2, \ldots$



NS Model provides a factor $1 - \alpha' p^2$, killing tachyon pole.
Vanishing of this diagram at $p = 0 \Rightarrow$ gluon massless.
All statements hold at finite $\alpha' > 0$.

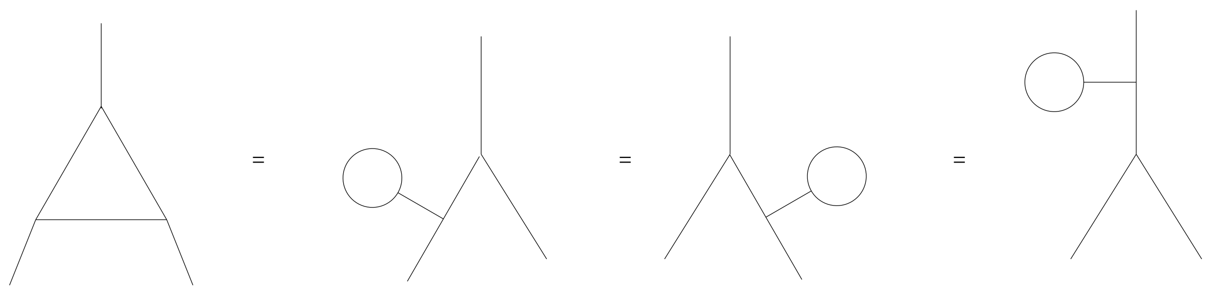
$$\mathcal{M}_2^+ \sim \int [dq]^+ \pi\alpha' (\epsilon_1 \cdot \epsilon_2 p^2 - 2p \cdot \epsilon_1 p \cdot \epsilon_2) \times \left[-\frac{1}{2} + \sum_{n=1}^{\infty} \frac{4q^{2n}}{(1-q^{2n})^2} + \sum_{r=1/2}^{\infty} \frac{4q^{2r}}{(1+q^{2r})^2} \right]$$

$$\mathcal{M}_2^- \sim \int [dq]^- \pi\alpha' (\epsilon_1 \cdot \epsilon_2 p^2 - 2p \cdot \epsilon_1 p \cdot \epsilon_2) \times \left[\sum_{n=1}^{\infty} \frac{4q^{2n}}{(1-q^{2n})^2} + \sum_{n=1}^{\infty} \frac{4q^{2n}}{(1+q^{2n})^2} \right]$$

Here $[dq]^\pm$ is shorthand for dq times
the θ independent factors of the integrands.

1-loop 3 Gluon Amplitude

Important point: this amplitude *includes both* 1-particle reducible and irreducible contributions:



Will normalization of reducible contributions be that of S-matrix or that of Green function?

(Recall S-matrix wave function factor in QFT is $Z^{n/2}$, whereas Green function supplies Z^n.)

Shall see GNS reg gives precisely S-matrix normalizaton!

Coefficient of $\epsilon_1 \cdot \epsilon_2 \sqrt{2\alpha'} k_1 \cdot \epsilon_3$ due to bosonic variables:

$$
\begin{aligned}
C^3_{\text{Bose}} \quad = \quad & \int [dq] \int_0^{2\pi} d\theta_3 \int_0^{\theta_3} d\theta_2 \\
& \left[\frac{1}{4} \csc^2 \frac{\theta_2}{2} - \sum_{n=1}^{\infty} n \frac{2q^{2n}}{1-q^{2n}} \cos n\theta_2 \right] \\
& \left[\frac{\sin(\theta_2/2)}{2\sin(\theta_3/2)\sin(\theta_{32}/2)} + \sum_{n=1}^{\infty} \frac{2q^{2n}}{1-q^{2n}} (\sin n\theta_{32} - \sin n\theta_3) \right] \\
& \psi(q,\theta_2)^{2\alpha' k_1 \cdot k_2} \psi(q,\theta_3)^{2\alpha' k_1 \cdot k_3} \psi(q,\theta_{32})^{2\alpha' k_2 \cdot k_3}
\end{aligned}
$$

The whole answer for bosonic string. Need fermi part for NS+

Metsaev and Tseytlin postulate: “1PIR amp” obtained by setting exponents to zero and replacing singular terms by their formal expansions

$$\frac{1}{4}\csc^2\frac{\theta}{2} \quad \to \quad -\sum_{n=1}^{\infty} n\cos n\theta, \qquad \frac{1}{2}\cot\frac{\theta}{2} \to \sum_{n=1}^{\infty}\sin n\theta$$

Then the θ integrals are elementary with the result

$$2\pi\sum_{n=1}^{\infty}\left(\frac{1+q^{2n}}{1-q^{2n}}\right)^2 \to 2\pi\left[-\frac{1}{2}+4\sum_{n=1}^{\infty}\frac{q^{2n}}{(1-q^{2n})^2}\right]$$

where the formal sum $\sum_n 1$ has been interpreted as $\zeta(0) = -1/2$. Prescriptions give the correct result in gauges where $Z_1 = Z_3$.

Unsatisfactory points:

- 1PIR not a Gauge Invariant concept: Need calculate **complete** contribution to S-matrix.
- More care needed for θ near $0, 2\pi$.

Handling θ divergences

Most singular part: Put

$$\begin{aligned} \alpha_1 &= 2\alpha' k_1 \cdot k_2, \qquad \alpha_2 = 2\alpha' k_1 \cdot k_3, \qquad \alpha_1 = 2\alpha' k_2 \cdot k_3 \\ k_i^2 &= 0, \qquad k_1 + k_2 + k_3 = p \end{aligned}$$

Then (Neveu and Scherk)

$$\frac{1}{2}\int_0^\pi d\theta_3 \int_0^{\theta_3} d\theta_2 [\sin\theta_2]^{\alpha_1 - 1}[\sin\theta_3]^{\alpha_2 - 1}[\sin\theta_{32}]^{\alpha_3 - 1} =$$

$$\frac{\sqrt{\pi}}{4}\frac{\Gamma(-(1-\alpha' p^2)/2)\Gamma(-\alpha_1/2)\Gamma(-\alpha_2/2)\Gamma(-\alpha_3/2)}{\Gamma(-(\alpha_1+\alpha_2)/2)\Gamma(-(\alpha_2+\alpha_3)/2)\Gamma(-(\alpha_3+\alpha_1)/2)}$$

$$\to +\frac{\pi}{2} = -\pi\left(1 - \frac{3}{2}\right), \qquad \text{for } p \to 0$$

Compared to $-\pi$ for corresponding term in “1PIR”

1PR contributions to $C^3_{\rm Bose}$

Contributions discarded in 1PIR formally $O(p^2)$, but enhanced by pole singularities from integration near $\theta_2, \theta_{32}, 2\pi - \theta_3 \sim 0$

E.g. integrating near $\theta_2 \sim 0$ gives

$$\frac{1}{2\alpha' k_1 \cdot k_2} \int_0^{2\pi} d\theta_3 \left[\frac{1}{4} \csc^2 \frac{\theta_3}{2} - \sum_{n=1}^{\infty} n \frac{2q^{2n}}{1-q^{2n}} \cos n\theta_3 \right]$$

$$\left[\sin \frac{\theta_3}{2} \prod_{n=1}^{\infty} \frac{(1-q^{2n}e^{i\theta_3})(1-q^{2n}e^{-i\theta_3})}{(1-q^{2n})^2} \right]^{2\alpha'(k_1+k_2)\cdot k_3}$$

$$\sim -\pi \frac{p \cdot k_3}{p \cdot k_3 + p^2/2} \left[-\frac{1}{2} + 4 \sum_{n=1}^{\infty} \frac{q^{2n}}{(1-q^{2n})^2} \right]$$

$$\to -\pi \left[-\frac{1}{2} + 4 \sum_{n=1}^{\infty} \frac{q^{2n}}{(1-q^{2n})^2} \right]$$

The other two singular regions give the same.

$$
\begin{aligned}
C^3_{\text{Bose}} &\to (2-3)\,\pi \int [dq] \left[-\frac{1}{2} + 4 \sum_{n=1}^{\infty} \frac{q^{2n}}{(1-q^{2n})^2} \right] \\
&\to -\pi \int [dq] \left[-\frac{1}{2} + 4 \sum_{n=1}^{\infty} \frac{q^{2n}}{(1-q^{2n})^2} \right]
\end{aligned}
$$

Note that agreement of the $\pi/2$ term with it's exact evaluation (explained earlier) confirms the interpretation of "$\sum_n 1$" as $\zeta(-1/2)$ by Metsaev and Tseytlin.

Contributions of the H correlators all notionally of order p^2, and require enhancement by pole singularities.

We simply quote the final result for fermi contributions:

$$C^{3,+}_{\rm Fermi} \quad\rightarrow\quad -\pi\int [dq]^{+}\left[4\sum_{r=1/2}^{\infty}\frac{q^{2r}}{(1+q^{2r})^{2}}\right]$$

$$C^{3,-}_{\rm Fermi} \quad\rightarrow\quad -\pi\int [dq]^{-}\left[\frac{1}{2}+4\sum_{n=1}^{\infty}\frac{q^{2n}}{(1+q^{2n})^{2}}\right]$$

and the total contributions:

$$C^{3,+}\rightarrow -\pi\int [dq]^{+}\left[-\frac{1}{2}+4\sum_{n=1}^{\infty}\frac{q^{2n}}{(1-q^{2n})^{2}}+4\sum_{r=1/2}^{\infty}\frac{q^{2r}}{(1+q^{2r})^{2}}\right]$$

$$C^{3,-}\rightarrow -\pi\int [dq]^{-}\left[4\sum_{n=1}^{\infty}\frac{q^{2n}}{(1-q^{2n})^{2}}+4\sum_{n=1}^{\infty}\frac{q^{2n}}{(1+q^{2n})^{2}}\right]$$

Quantities in $[\cdots]$ identical to those in 2 gluon function!

Field Theory Limit

Need q integration near $q \sim 1$ or $w < \exp(-1/\alpha'\Lambda^2)$

$$
\begin{aligned}
[dq]^{\pm} \quad &= \quad \frac{2(g\sqrt{2\alpha'})^3}{(8\pi^2\alpha')^{D/2}} \frac{1 \mp \sqrt{w}}{\sqrt{w}} \frac{\prod(1 \pm w^r)^{D-1}}{(1-w^n)^{D-1}} \left(-\frac{2\pi}{\ln w}\right)^{1+D/2} \frac{dw}{2w} \\
&\sim \quad \frac{2(g\sqrt{2\alpha'})^3}{(8\pi^2\alpha')^{D/2}} \frac{1 \pm (D-2)\sqrt{w}}{\sqrt{w}} \left(-\frac{2\pi}{\ln w}\right)^{1+D/2} \frac{dw}{2w}
\end{aligned}
$$

Last line is valid for $w \ll 1$. Compare bosonic string measure:

$$
[dq]^B \sim \frac{2(g\sqrt{2\alpha'})^3}{(8\pi^2\alpha')^{D/2}} \frac{1+(D-2)w}{w} \left(-\frac{2\pi}{\ln w}\right)^{1+D/2} \frac{dw}{2w}
$$

in the same limit.

Also need:

$$4\sum_n \frac{q^{2n}}{(1-q^{2n})^2} \quad \sim \quad \frac{1}{6} + \frac{\ln w}{2\pi^2} + \frac{\ln^2 w}{24\pi^2} - w\frac{\ln^2 w}{\pi^2} + O(w^2)$$

$$4\sum_r \frac{q^{2r}}{(1+q^{2r})^2} \quad \sim \quad -\frac{\ln w}{2\pi^2} - w^{1/2}\frac{\ln^2 w}{\pi^2} + O(w)$$

$$4\sum_n \frac{q^{2n}}{(1+q^{2n})^2} \quad \sim \quad -\frac{1}{2} - \frac{\ln w}{2\pi^2} + w^{1/2}\frac{\ln^2 w}{\pi^2} + O(w)$$

Which enter in the combinations:

$$-\frac{1}{2} + 4\sum_{n=1}^{\infty} \frac{q^{2n}}{(1-q^{2n})^2} + 4\sum_{r=1/2}^{\infty} \frac{q^{2r}}{(1+q^{2r})^2} \sim -\frac{1}{3} + \left[\frac{1}{24\pi^2} - \frac{\sqrt{w}}{\pi^2}\right]\ln^2 w$$

$$4\sum_{n=1}^{\infty} \frac{q^{2n}}{(1-q^{2n})^2} + 4\sum_{r=1/2}^{\infty} \frac{q^{2r}}{(1+q^{2r})^2} \sim -\frac{1}{3} + \left[\frac{1}{24\pi^2} + \frac{\sqrt{w}}{\pi^2}\right]\ln^2 w$$

$$
\begin{aligned}
\frac{C_3^+ - C_3^-}{2} \quad &\sim \quad -\pi \frac{(g\sqrt{2\alpha'})^3}{(8\pi^2\alpha')^{D/2}} \int_\epsilon^{\exp(-1/\alpha'\Lambda^2)} \frac{dw}{2w} \left(-\frac{2\pi}{\ln w}\right)^{1+D/2} \\
&\left(\frac{1+(D-2)\sqrt{w}}{\sqrt{w}} \left\{ -\frac{1}{3} + \left[\frac{1}{24\pi^2} - \frac{\sqrt{w}}{\pi^2}\right] \ln^2 w \right\} \right. \\
&\left. - \frac{1-(D-2)\sqrt{w}}{\sqrt{w}} \left\{ -\frac{1}{3} + \left[\frac{1}{24\pi^2} + \frac{\sqrt{w}}{\pi^2}\right] \ln^2 w \right\} \right) \\
&\sim \quad \pi \frac{(g\sqrt{2\alpha'})^3}{(8\pi^2\alpha')^{D/2}} \int_\epsilon^{\exp(-1/\alpha'\Lambda^2)} \frac{dw}{w} \left(-\frac{2\pi}{\ln w}\right)^{1+D/2} \\
&\left(\frac{D-2}{3} - \frac{D-26}{24\pi^2} \ln^2 w \right)
\end{aligned}
$$

1PIR part (a la Metsaev/Tseytlin) of $(D-26)$ is $-2(D-2)$

$$1\text{PR} = D - 26 + 2(D-2) = 3(D-10)$$

Put $\ln w = -1/\alpha'\lambda$, $dw/w = d\lambda/\alpha'\lambda^2$, $\ln \epsilon = -1/\alpha'\lambda_0$:

$$\begin{aligned} \frac{C_3^+ - C_3^-}{2} &\sim \frac{g^3\,(4\pi)^{2-D/2}}{2\sqrt{2\alpha'}} \int_{\lambda_0}^{\Lambda^2} \frac{d\lambda}{\lambda} \\ &\quad \left(\frac{D-2}{3} \alpha'^2 \lambda^{D/2} - \frac{D-26}{24\pi^2} \lambda^{(D-4)/2} \right) \\ &\sim \frac{g^3}{2\sqrt{2\alpha'}} \left(\frac{1}{3} \alpha'^2 \Lambda^4 + \frac{22}{24\pi^2} \ln \frac{\Lambda^2}{\lambda_0} \right) \end{aligned}$$

for $D = 4$. The tree contribution to this quantity is $2g/\sqrt{2\alpha'}$

$$\begin{aligned} g_R &= g\left(1 + \frac{11 g^2}{48\pi^2} \ln \frac{\Lambda^2}{\lambda_0} + O(g^4)\right) \\ \frac{N_c \alpha_s(\lambda_0)}{\pi} &= \frac{g_R^2}{2\pi^2} = \frac{12}{11 \ln \lambda_0/M^2} + O\left(\frac{1}{\ln^2 \lambda_0/M^2}\right) \end{aligned}$$

Summary

- Studied field theory limit of 1-loop NS+ model
- UV cutoff on q: $-\ln q < 2\pi^2\alpha'\Lambda$
- GNS regularization, used throughout, consistently handles all the formally IR divergent θ integrations.
- Gluon self-energy vanishes and coupling renomalization correctly given
- This study is preliminary to setting up a systematic formalism to sum planar open string diagrams by, for example, employing the lightcone worldsheet.