Large N Gauge Theory from Open String Worldsheets

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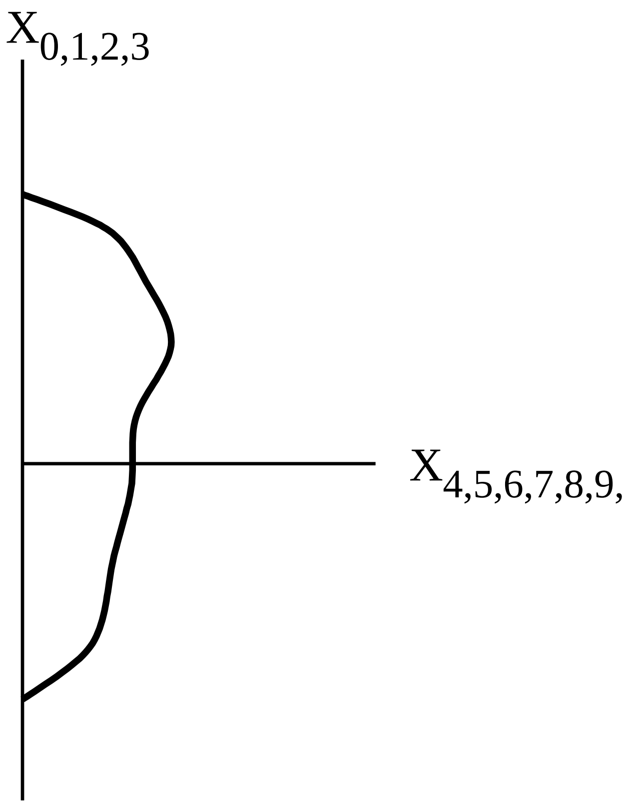
String Basis for Field/String Duality

Open String with SU(N) Chan-Paton $\underset{\alpha' \to 0}{\Longrightarrow}$ SU(N) Yang-Mills (Scherk, Neveu and Scherk, 1971)

Left side a regulated version of right side

Open String Trees $\Rightarrow$ All String Tree and Loop Diagrams ($\alpha' > 0$) (1970)

D3-branes and 4D QFT



$$x^M(\sigma,\tau) : \begin{cases} \text{Neumann b.c.}'\text{s} & M = 0,1,2,3 \equiv \mu \\ \text{Dirichlet b.c.}'\text{s} & M = 4,5,6,7,8,9 \end{cases}$$

't Hooft's $N \to \infty$:

$$\sum(\text{Planar Open String Loops})_{D3} \equiv \sum(\text{Closed String Trees})_{D3\text{source}}$$

Left Side $\underset{\alpha' \to 0}{\Longrightarrow}$ $N = \infty$ Gauge Theory in 4d

Right Side $\underset{\substack{\alpha' \to 0 \\ g^2 N \to \infty}}{\Longrightarrow}$ Classical gravity

If $g^2 N = O(1)$, right side stays stringy as $\alpha' \to 0$.
I.e. must solve classical closed string field eqs.

Theme of this talk:

$N \to \infty$ QCD by direct planar graph summation at $\alpha' > 0$.

String dual for given QFT

AdS/CFT Paradigm:

Lift $\mathcal{N} = 4$ Yang-Mills to NSR/GSO Open String ending on D3-branes in 10D Minkowski space-time

Bulk of open string vibrates in all 10 space-time dimensions.

Massless states:

- an adjoint vector: vibrations $\parallel$ D3-branes,
- 6 adjoint scalars: vibrations $\perp$ D3-branes,
- 4 Majorana fermions.

Simplest Open String for Pure 4D Yang-Mills

Delete fermionic states (no R sector)

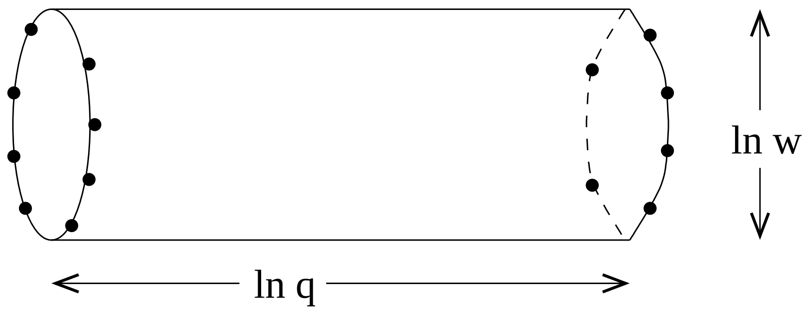
Even G-parity sector of Neveu-Schwarz (NS+) open string

Simplest choice for YM: NS+ model in 4D:

- Trees are physically sound (no ghosts or tachyons)
  $\rightarrow$ YM trees at low energy

- >2 Loops still poorly understood

1 Loop from Unitarity: closed string spectrum



- $q^{-(D-1)/8} \rightarrow q^{-(D-1)/8+\alpha' Q^2/2}$

- $\mathcal{M}_{\rm NP}^{+}$ has closed string cut starting at $\alpha' Q^2 = \frac{D-1}{4}$, but $\mathcal{M}_{\rm NP}^{-}$ closed string cut starts at $Q^2 = 0$.

- Closed string has continuous mass spectrum

Interpreting the closed string cut: Holography

Factors not present for critical superstring $D = 10$:

$$\sqrt{\frac{-\pi}{\ln q}}(w^{(D-9)/16} \mp w^{(D-1)/16})$$

$$= \int dK q^{K^2/4} \begin{cases} \sin\frac{K\gamma_+}{2} \sin\frac{K\gamma_-}{2} \\ \cos\frac{K\gamma_+}{2} \cos\frac{K\gamma_-}{2} \end{cases}$$

$$\gamma_\pm = \sqrt{\frac{D-1}{16}} \pm i\sqrt{\frac{9-D}{16}}$$

K: the momentum of an extra dimension

Open strings are "Dp-branes" in $D+1$ dimensional closed string theory, with $p = D - 1$.

For $D < 9$, Dp-branes located at complex values, $\pm\gamma_+/2$ and $\pm\gamma_-/2$, of the holographic dimension!

Holographic low energy closed string spectrum

- Tachyon pole at $\alpha'(Q^2 + K^2)/2 = (D-1)/8$ in $\mathcal{M}^+$.
- No massless graviton poles
- Massless RR (5D) closed string states in $\mathcal{M}^-$

$$S, \qquad A_{\mu\nu}, \qquad A_\mu$$

- Planar diagram sum should resolve the IR issues connected to the tachyon and the RR massless states.

No extra scalars and massive would-be graviton –but ...

$D = 4 < 9 \Rightarrow$ technical complications (complex holographic coordinates associated with conformal anomaly) in closed sector

Keeping $D = 10$ (or $D = 9$) $\Rightarrow$ conformal anomaly cancels.

But: D3-brane trick $\Rightarrow$ 6 (or 5) massless scalars in the 4D theory.

$D = 9$ removes 1 massless scalar – Let's understand it better!

Consider more carefully NS open string in $D = 9$:

$$Z(w) = (1 - w^{1/2}) \frac{\prod_r (1 + w^r)^8}{\prod_n (1 - w^n)^8}$$

Compared to 10D NS open string ending on a D8-brane:

$$Z(w) = \frac{\prod_r (1 + w^r)^8}{\prod_n (1 - w^n)^8}$$

First case: 7 massless states (9D gluon)

Second case: 8=7+1 massless states (9D gluon + 1 scalar)

Goal: Modify D8 b.c.'s to achieve same spectrum

T-dual D-brane conditions

For a D8-brane at $x^9 = 0$:

$$x^9(0,\tau) \quad = \quad x^9(\pi,\tau) = 0$$

T-dual transform $x^9(\sigma,\tau) \rightarrow y^9(\sigma,\tau)$:

$$\frac{\partial y^9}{\partial \sigma}(0,\tau) = \frac{\partial y^9}{\partial \sigma}(\pi,\tau) = 0.$$

Then the zero mode of y^9:

$$p_0^9 \equiv \int d\sigma \dot{y}^9(\sigma,\tau) = x^9(\pi,\tau) - x^9(0,\tau) = 0$$

SU(2) Invariance

Interpret y^9 as a $c = 1$ conformal scalar field,
compactified on a circle: $p_0^9 = 2\pi n/R$.

SU(2) symmetry emerges when R is such that
$|0, \pm 2\pi/R\rangle$ are massless.
$(b^9_{-1/2}|0\rangle, |0, \pm 2\pi/R\rangle)$ transform as a vector

Invariance under SU(2) $\Rightarrow n = 0$ **and** projects out $b^9_{-1/2}|0\rangle$

Why not repeat for x^8, x^7, x^6, x^5, x^4:

SU(2) invariance for each of 6 extra coordinates,
projects out all massless scalars in open string state space.

SU(2) symmetry, NS Open String

Set $J_3 = p_0^I \sqrt{2\alpha'}$, SU(2) character on state space:

$$
\begin{aligned}
X_{\rm NS}(\theta) &\equiv {\rm Tr} w^{L_0} e^{-i\theta J_3} \\
&= \sum_{k=-\infty}^{\infty} w^{k^2/2} e^{ik\theta} \prod_{r=1/2}^{\infty} (1+w^r) \prod_{n=1}^{\infty} \frac{1}{1-w^n} \\
&= \sum_{k=0}^{\infty} w^{k^2/2} \chi_k(\theta) \prod_{r\neq k+1/2}^{\infty} (1+w^r) \prod_{n\neq 2k+1}^{\infty} \frac{1}{1-w^n} \\
\chi_j(\theta) &\equiv \sum_{m=-j}^{j} e^{im\theta}
\end{aligned}
$$

Here χ_j is the character for the spin j irreducible representation of SU(2).

Comments

- Spinor representations of SU(2) are absent (they were in the deleted Ramond sector!),
- Symmetry is O(3).
- Partition function for O(3) invariant subspace:

$$\prod_{r=3/2}^{\infty} (1 + w^r) \prod_{n=2}^{\infty} \frac{1}{1 - w^n},$$

- Absence of $r = 1/2$ reflects absence of massless scalar

Vertex operator construction of SU(2) generators

$$
\begin{aligned}
J_3 &= p_0^I \sqrt{2\alpha'} \\
J_\pm &= \sqrt{2} \oint \frac{dz}{2\pi i z} H^I(z) : e^{\pm i y^I(z)/\sqrt{2\alpha'}} :
\end{aligned}
$$

$: e^{i y^I(z)/\sqrt{2\alpha'}} :$ is a bosonized fermionic field, when acting on the states with $p_0^I \in \mathbb{Z}/\sqrt{2\alpha'}$.

$[J_+, J_-] = 2J_3$.

Essential point:

$[J_\pm, G_r] = [J_\pm, L_n] = 0$ on this subspace,

so the SU(2) commutes with physical state conditions

Manifestly O(3) Invariant Description

Replace each bosonic y^I with a pair of fermion fields H_1^I, H_2^I

Call original $H^I \equiv H_3^I$.

Then H_a transform as a vector under $O(3)$ with generators

$$J^a = \epsilon^{abc} \oint \frac{dz}{2\pi i z} H_b(z) H_c(z).$$

Nonabelian D-brane condition: $J^a |\mathrm{Phys}\rangle = 0$.

Scattering Amplitudes

Trees are subset of 10D NS trees:

Take external strings only in even G-parity states invariant under 6 SU(2)'s associated with the 6 extra dimensions.

Noninvariant states decouple, including the massless scalars:

$b^I_{-1/2}|0, p\rangle$, $I = 4, 5, \ldots, 9$

One Loop and Closed Strings

Loops require projectors onto SU(2) invariant states.

At one loop simply multiplies partition function by $(1 \mp w^{1/2})^6$
Also loop momentum integral only over p^μ, $\mu = 0, 1, 2, 3$

After Jacobi transformation to cylinder variables $\ln q = \frac{2\pi^2}{\ln w}$, these differences produce the extra factors

$$\left[\sqrt{\frac{-\pi}{\ln q}}(1 \mp w^{1/2})\right]^6 = \begin{cases} \int d^6 K q^{K^2/4} \prod_{I=4}^{9} \sin^2 \frac{K^I}{2\sqrt{2}} \\ \int d^6 K q^{K^2/4} \prod_{I=4}^{9} \cos^2 \frac{K^I}{2\sqrt{2}} \end{cases}$$

relative to the usual one loop integrand in $D = 10$.

Closed String Coupling to Nonabelian D3-Branes

Difference from the normal D-brane integrand is factor $\psi^2_\pm(K^I)$

$$\psi_+(K^I) = 2^3 \prod_{I=4}^{9} \sin(K^I/2\sqrt{2}), \quad \psi_-(K^I) = 2^3 \prod_{I=4}^{9} \cos(K^I/2\sqrt{2})$$

The $+$ and $-$ refer to the NS-NS and R-R sectors, respectively, of the type-0 closed string system.

Factor $\psi_\pm$ introduces an asymmetry in the coupling of these two sectors compared to the normal D-brane.

NS-NS suppressed relative to R-R at low energy $(K \ll 1)$

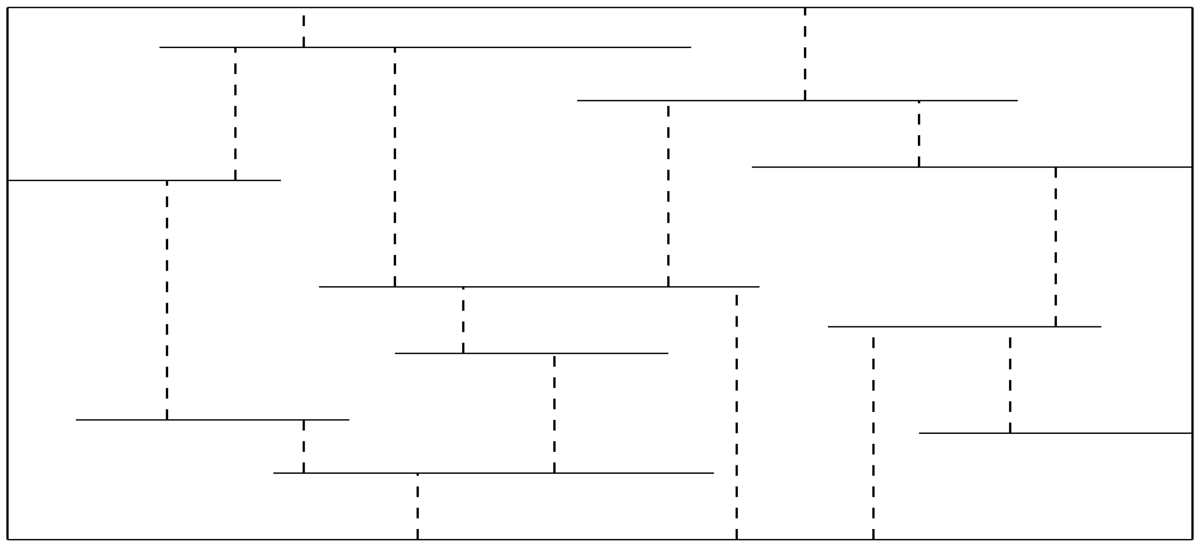
Type-0 Closed Strings

The closed string system that propagates in the 10 dimensional bulk is the critical type-0 closed string, which is well-studied.

The coupling of these closed strings to a nonabelian D3-brane is significantly different from the coupling to the normal (abelian) D3-brane.

It is essential that the path integrand be local. Normal D-brane conditions are applied locally on the worldsheet $x_{\rm ends} = 0$. But this condition looks non-local in T-dual variables $\int d\sigma \dot{y} = 0$, which were used to formulate nonabelian D-brane conditions.

Need to consider them a little more carefully.



Projector for the Ith SU(2):

$$P_I = \int dR e^{i\theta_a J_I^a}, \qquad J_I^a = \int d\sigma \mathcal{J}_I^a(\sigma)$$

dR is the O(3) invariant Haar measure.

The number of such projector $P = \prod_{I=4}^{9} P_I$ insertions changes with the appearance or disappearance of a horizontal line.

Gauging the O(3)'s

Since $P^2 = P$ we can put a projector on each time-slice of each string propagator. Introduce independent R's for each point σ, t:

$$\begin{aligned} P_I &= \prod_t P_I = \int \prod_t dR(t) e^{i\theta_a(t) J_I^a} \\ &= \int \prod_t \prod_\sigma dR(\sigma, t) e^{i \int d\sigma \theta_a(\sigma,t) \mathcal{J}_I^a} \delta(R'(\sigma, t)) \end{aligned}$$

Delete the $\delta(R'(\sigma, t))$ factor whenever σ sits on a horizontal line.

We have gauged each O(3) symmetry on the worldsheet, replacing the factor $e^{-\int F^2/4}$ with $\prod_{\sigma,t} \delta(F(\sigma, \tau))$.

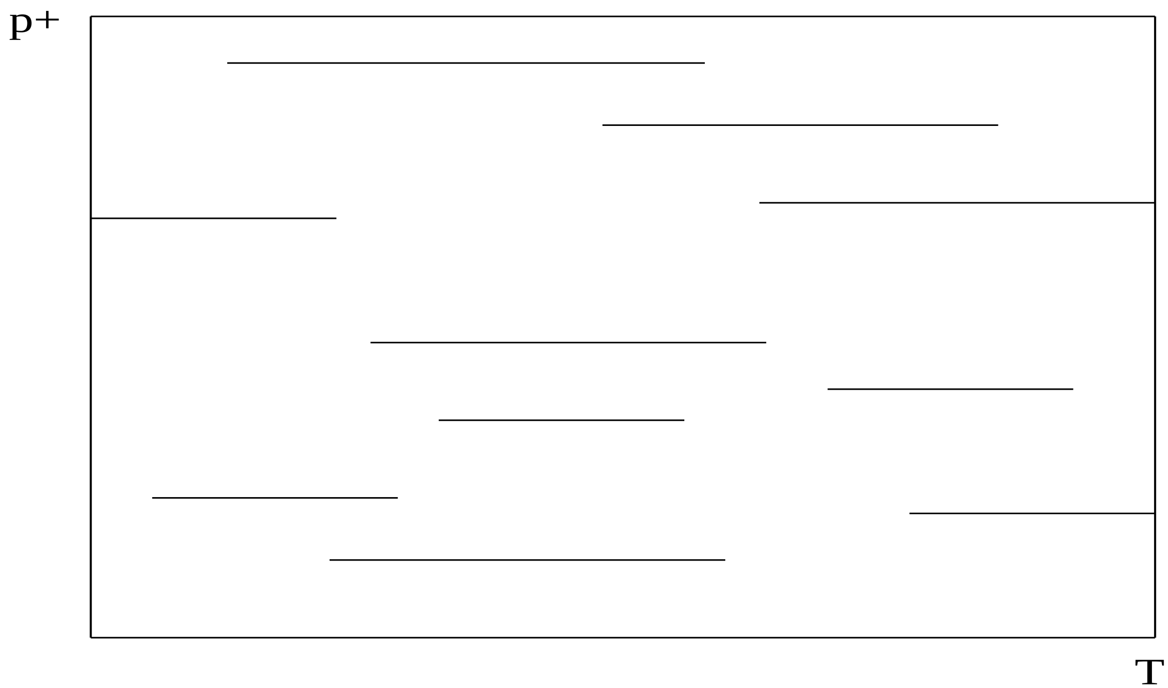
Worldsheet local prescription for inserting projectors.

Summing Planar Diagrams on the Lightcone

Mandelstam's lightcone path integral formalism provides a systematic approach to this problem

Potentially an effective numerical assault on the problem

Mandelstam Interacting String Diagram:



$$T = ix^{+} = i(t+z)/\sqrt{2}, \qquad p^{+} = (p^{0} + p^{z})/\sqrt{2}.$$

Diagram describes time evolution of a system of open strings, breaking and rejoining as shown by the horizontal lines.

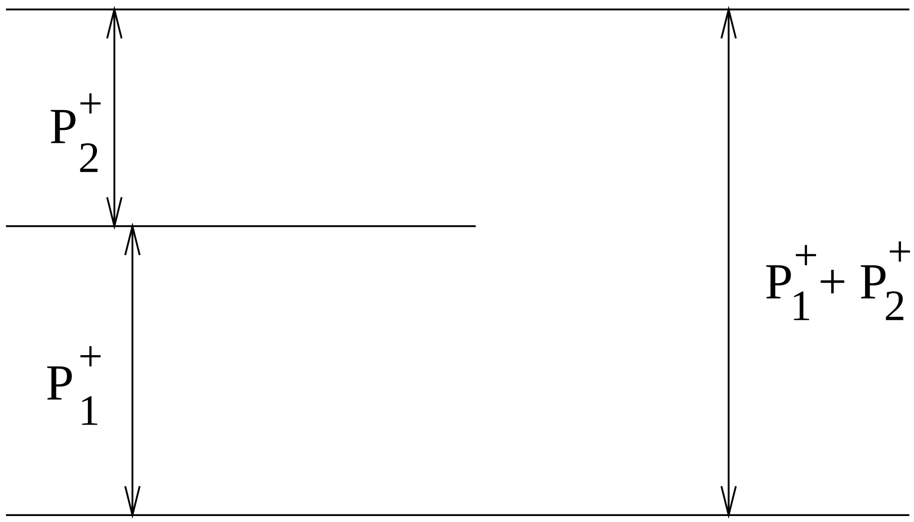
For **critical** open string, worldsheet path integral uses lightcone action for the free open string:

$$S_{l.c.} = \frac{1}{2}\int_0^T d\tau \int_0^{P^+} d\sigma \left[\left(\frac{\partial \boldsymbol{x}}{\partial \tau}\right)^2 + T_0^2 \left(\frac{\partial \boldsymbol{x}}{\partial \sigma}\right)^2\right]$$

$\sum$(planar diagrams)= $\sum$(#, length, location of horizontal lines).

For each beginning and end of a horizontal line there is a factor of string coupling $g\times$(prefactor).

Normalization



Lorentz covariance: Vertex $\sim \frac{1}{\sqrt{P_1^+ P_2^+ (P_1^+ + P_2^+)}}$

So under $P_i^+ \to \lambda P_i^+$, above diagram should scale as $\lambda^{-3/2}$.

Worldsheet Lattice calculation gives $\lambda^{-(D-2)/16}$ for bosonic string
$D = 26$ for Lorentz covariance (Giles-CBT,1977)

Hearing the Shape of a (Polygonal) Drum
(Mark Kac, 1967)

$$\mathrm{Tr}\ e^{t\nabla^2/2} \sim \frac{\mathrm{Area}}{2\pi t} - \frac{\mathrm{Perimeter}}{4\sqrt{2\pi t}} + \sum_{\mathrm{corners}} \frac{1}{24}\left(\frac{\pi}{\theta} - \frac{\theta}{\pi}\right) + o(1)$$

Lightcone vertex is a 360° corner. Putting $\theta = 2\pi$,

$$\frac{1}{24}\left(\frac{\pi}{\theta} - \frac{\theta}{\pi}\right) \rightarrow -\frac{1}{16}$$

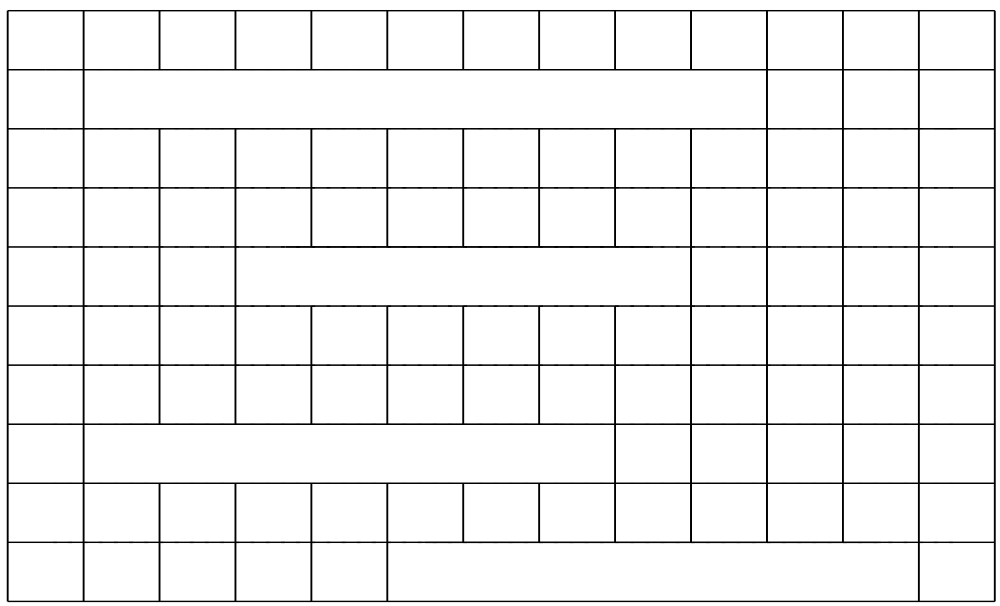
Notice that rounding the corner would spoil this nice result

A case where smoother is NOT better!

Lightcone Worldsheet for Planar Sum

Mark the presence or absence of a horizontal line at any point by an Ising spin variable $S(\sigma, \tau) = 1, 0$.

Worldsheet Lattice (Bosonic String, Giles-CBT, 1977):



$$
\begin{aligned}
S \quad \rightarrow \quad & \frac{1}{2}\sum_{ij}\left[(x_i^{j+1}-x_i^j)^2+T_0^2 S_i^j(x_{i+1}^j-x_i^j)^2\right] \\
& -\sum_{ij}\left[S_i^j(1-S_i^{j+1})+S_i^{j+1}(1-S_i^j)\right]\ln g
\end{aligned}
$$

Monte Carlo simulations very feasible for this system

Caveat: This lattice system reproduces, at each loop order, an integral over moduli space that is formally divergent (because of tachyons in the bosonic open string).

Goddard/Neveu-Scherk Regularization: Give momentum to each hole in the worldsheet then continue to 0 momentum.

NS+ model has no open string tachyons. Closed string tachyons associated with $w \rightarrow 1$ may be resolved by planar graph sum.

GNS Example: Open String Self Energy

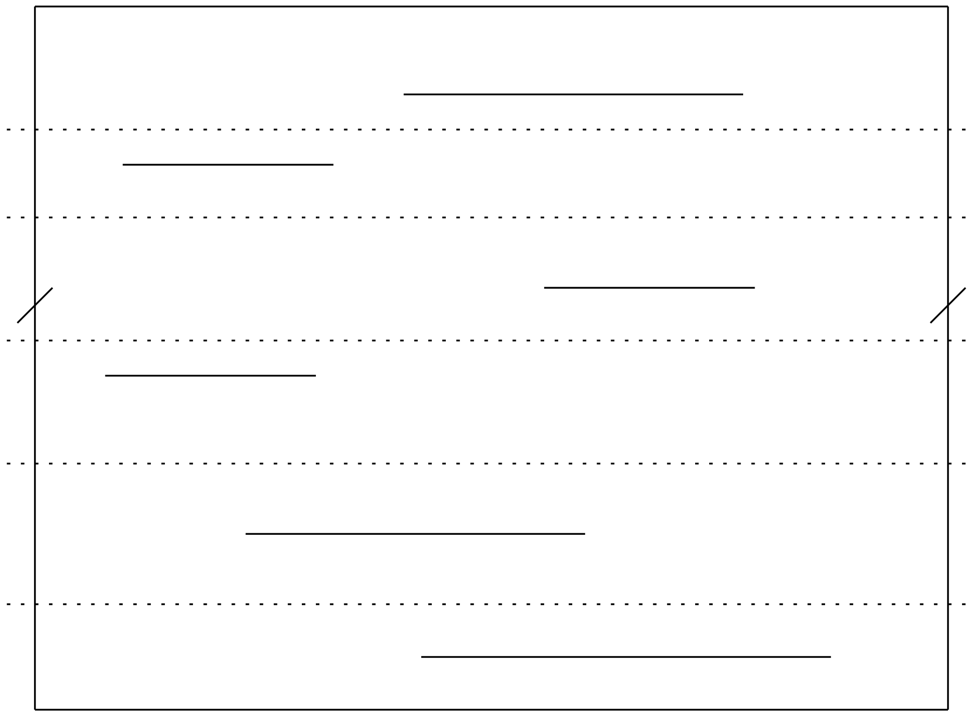
$$\Sigma_{\text{Gluon}} \sim \frac{1}{2}\int_0^{\pi} d\theta (\sin\theta)^{2\alpha' k_1\cdot k_2 - 2}$$

Now, $k_1^2 = k_2^2 = 0$. GNS: $k_1 + k_2 \equiv p$, with $p \to 0$ at end.

$$\begin{aligned}\Sigma_{\text{Gluon}} \to \frac{1}{2}\int_0^{\pi} d\theta (\sin\theta)^{\alpha' p^2 - 2} &= \frac{1}{2}\frac{\Gamma(1/2)\Gamma(-1/2+\alpha' p^2/2)}{\Gamma(\alpha' p^2/2)} \\ &\to -\frac{\pi\alpha' p^2}{2} \to 0\end{aligned}$$

So the gluon remains massless!

Alternate Approach to Planar Sum (Kruczenski)



A geometric sum: adds a hole operator to the free closed string Hamiltonian.

Summary

- Open String ($\alpha' > 0$) determines/regulates gauge theory
- Open/Closed Duality $\underset{\alpha' \to 0}{\Longrightarrow}$ Field/String Duality
- Open String for 4D Yang-Mills:
  10D NS+ with nonabelian D3-brane b.c.'s on 6 dimensions
- Lightcone path integrals: a potential non-perturbative formulation for $\sum$(Planar Diagrams).
  - Requires careful assessment of the fidelity of its representation of moduli integrals at each loop order.
  - Contact terms may be needed
  - Work in progress.

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