Standard Model II

Take Home Final Examination

24 April 2020, 5pm

Due via email: 27 April 2020 at noon (12:01 pm)

INSTRUCTIONS

1. This exam consists of a series of 10 short answer/mini-essay questions, worth 10 points each.
2. No collaboration allowed. You may consult our lecture notes, posted homework solutions, and your favorite textbooks on QFT and/or Standard Model. But all reasoning and explanation must be in your own words.
3. No points will be deducted if purely numerical factors like powers of 2, π, etc. are missed in definitions or derivations.
4. Please email me if clarifications are needed. I will respond via an email to the whole class, so that everyone will have the same information.

1. Before the charm quark was discovered, a serious challenge to our understanding of weak interactions was the extreme suppression of the weak decay $K_L^0 \to \mu^+\mu^-$. The rate was nonzero but several orders of magnitude smaller than expected from the one loop diagram. Explain how the existence of the charm quark resolves this problem to one loop order. (Note that it is not enough to show suppression at tree level!)

2. What is the $U(1)$ problem in chiral dynamics and how do anomalous triangle diagrams, together with instantons, resolve it?

3. In the minimal standard model, with only a complex doublet Higgs field, the Higgs potential has a larger symmetry than $SU(2)\times U(1)$. Identify this larger symmetry and explain its implications for the relative strength of charged current and neutral current interactions in the low energy effective Lagrangian.

4. Draw the box diagrams in the standard model that produce $K^0 - \bar{K}^0$ mixing, and estimate (only in order of magnitude!) the mass difference it predicts.

5. Explain why the β decay $\pi^- \to \pi^0 + e^- + \bar{\nu}_e$ is a good way to measure the element V_{ud} of the CKM quark mixing matrix.

6-8. The low energy weak interactions of nucleons and pions are largely explained in terms of two parameters g_A and F_π.

6) Explain how these two parameters are defined,

7) Derive the relation $G_{\pi N}F_\pi = m_N g_A$ (Goldberger-Treiman), where $G_{\pi N}$ is the pion nucleon coupling.

8) What does this relation have to do with spontaneous symmetry breaking? In your answer, make sure you clearly identify the symmetry that is broken!

9. In addition to the result of 7 above, there is a second relation (Adler-Weisberger). The two relations together then determine g_A, F_π in terms of strong interaction physics. Without going into technical details, describe (verbally) the methodology used to obtain this second relation, making sure to specify the assumptions made.

10. The three light quarks, u, d, and s, enjoy an approximate global $SU(3)$ "flavor" symmetry in QCD, broken only by the inequality of their masses and charges. The flavor symmetry breaking terms in the Standard Model Lagrangian transform as the "3" and "8" components of an $SU(3)$ octet. To predict the mass patterns within the baryon octet and decuplet to first order in the breaking, one therefore needs the matrix elements of a generic octet operator $\langle r|\Omega_a|s\rangle$ where r, s label states in the **same** irreducible $SU(3)$ representation. For $SU(2)$ in this situation the Wigner Ekart theorem simplifies to the statement that any vector operator can be replaced by a numerical multiple of the angular momentum operator in such matrix elements. What is the corresponding simplification for the matrix elements in the $SU(3)$ case?