standing on the ground (which we shall take to be an inertial frame) beside a perfectly flat horizontal turntable, rotating with constant angular velocity ω. I lean over and shove a frictionless puck so that it slides across the turntable, straight through the center. The puck is subject to zero net force and, as seen from my inertial frame, travels in a straight line. Describe the puck's path as observed by someone sitting at rest on the turntable. This requires careful thought, but you should be able to get a qualitative picture. For a quantitative picture, it helps to use polar coordinates; see Problem 1.46. A

SECTION 1.5 The Third Law and Conservation of Momentum

1.28 ⋆ Go over the steps from Equation (1.25) to (1.29) in the proof of conservation of momentum, but treat the case that $N = 3$ and write out all the summations explicitly to be sure you understand the various manipulations.

1.29 ⋆ Do the same tasks as in Problem 1.28 but for the case of four particles ($N = 4$).

1.30 ⋆ Conservation laws, such as conservation of momentum, often give a surprising amount of information about the possible outcome of an experiment. Here is perhaps the simplest example: Two objects of masses m_1 and m_2 are subject to no external forces. Object 1 is traveling with velocity $\mathbf{v}$ when it collides with the stationary object 2. The two objects stick together and move off with common velocity $\mathbf{v}'$. Use conservation of momentum to find $\mathbf{v}'$ in terms of $\mathbf{v}$, m_1, and m_2.

1.31 ⋆ In Section 1.5 we proved that Newton's third law implies the conservation of momentum. Prove the converse, that if the law of conservation of momentum applies to every possible group of particles, then the interparticle forces must obey the third law. [*Hint:* However many particles your system contains, you can focus your attention on just two of them. (Call them 1 and 2.) The law of conservation of momentum says that if there are no external forces on this pair of particles, then their total momentum must be constant. Use this to prove that $\mathbf{F}_{12} = -\mathbf{F}_{21}$.]

1.32 ⋆⋆ If you have some experience in electromagnetism, you could do the following problem concerning the curious situation illustrated in Figure 1.8. The electric and magnetic fields at a point $\mathbf{r}_1$ due to a charge q_2 at $\mathbf{r}_2$ moving with constant velocity $\mathbf{v}_2$ (with $v_2 \ll c$) are[15]

$$\mathbf{E}(\mathbf{r}_1) = \frac{1}{4\pi\epsilon_0}\frac{q_2}{s^2}\hat{\mathbf{s}} \quad \text{and} \quad \mathbf{B}(\mathbf{r}_1) = \frac{\mu_0}{4\pi}\frac{q_2}{s^2}\mathbf{v}_2 \times \hat{\mathbf{s}}$$

where $\mathbf{s} = \mathbf{r}_1 - \mathbf{r}_2$ is the vector pointing from $\mathbf{r}_2$ to $\mathbf{r}_1$. (The first of these you should recognize as Coulomb's law.) If $\mathbf{F}_{12}^{\text{el}}$ and $\mathbf{F}_{12}^{\text{mag}}$ denote the electric and magnetic forces on a charge q_1 at $\mathbf{r}_1$ with velocity $\mathbf{v}_1$, show that $F_{12}^{\text{mag}} \le (v_1 v_2/c^2) F_{12}^{\text{el}}$. This shows that in the non-relativistic domain it is legitimate to ignore the magnetic force between two moving charges.

1.33 ⋆⋆⋆ If you have some experience in electromagnetism and with vector calculus, prove that the magnetic forces, $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$, between two steady current loops obey Newton's third law. [*Hints:* Let the two currents be I_1 and I_2 and let typical points on the two loops be $\mathbf{r}_1$ and $\mathbf{r}_2$. If $d\mathbf{r}_1$ and $d\mathbf{r}_2$ are short segments of the loops, then according to the Biot–Savart law, the force on $d\mathbf{r}_1$ due to $d\mathbf{r}_2$ is

$$\frac{\mu_0}{4\pi}\frac{I_1 I_2}{s^2} d\mathbf{r}_1 \times (d\mathbf{r}_2 \times \hat{\mathbf{s}})$$

where $\mathbf{s} = \mathbf{r}_1 - \mathbf{r}_2$. The force $\mathbf{F}_{12}$ is found by integrating this around both loops. You will need to use the "$BAC - CAB$" rule to simplify the triple product.]

[15] See, for example, David J. Griffiths, *Introduction to Electrodynamics*, 3rd ed., Prentice Hall, (1999), p. 440.

1.34 ⋆⋆⋆ Prove that in the absence of external forces, the total *angular* momentum (defined as $\mathbf{L} = \sum_\alpha \mathbf{r}_\alpha \times \mathbf{p}_\alpha$) of an N-particle system is conserved. [*Hints*: You need to mimic the argument from (1.25) to (1.29). In this case you need more than Newton's third law: In addition you need to assume that the interparticle forces are *central*; that is, $\mathbf{F}_{\alpha\beta}$ acts along the line joining particles α and β. A full discussion of angular momentum is given in Chapter 3.]

SECTION 1.6 Newton's Second Law in Cartesian Coordinates

1.35 ⋆ A golf ball is hit from ground level with speed v_0 in a direction that is due east and at an angle θ above the horizontal. Neglecting air resistance, use Newton's second law (1.35) to find the position as a function of time, using coordinates with x measured east, y north, and z vertically up. Find the time for the golf ball to return to the ground and how far it travels in that time. A

1.36 ⋆ A plane, which is flying horizontally at a constant speed v_0 and at a height h above the sea, must drop a bundle of supplies to a castaway on a small raft. **(a)** Write down Newton's second law for the bundle as it falls from the plane, assuming you can neglect air resistance. Solve your equations to give the bundle's position in flight as a function of time t. **(b)** How far before the raft (measured horizontally) must the pilot drop the bundle if it is to hit the raft? What is this distance if $v_0 = 50$ m/s, $h = 100$ m, and $g \approx 10$ m/s^2? **(c)** Within what interval of time ($\pm\Delta t$) must the pilot drop the bundle if it is to land within ± 10 m of the raft?

1.37 ⋆ A student kicks a frictionless puck with initial speed v_0, so that it slides straight up a plane that is inclined at an angle θ above the horizontal. **(a)** Write down Newton's second law for the puck and solve to give its position as a function of time. **(b)** How long will the puck take to return to its starting point? A

1.38 ⋆ You lay a rectangular board on the horizontal floor and then tilt the board about one edge until it slopes at angle θ with the horizontal. Choose your origin at one of the two corners that touch the floor, the x axis pointing along the bottom edge of the board, the y axis pointing up the slope, and the z axis normal to the board. You now kick a frictionless puck that is resting at O so that it slides across the board with initial velocity $(v_{0x}, v_{0y}, 0)$. Write down Newton's second law using the given coordinates and then find how long the puck takes to return to the floor level and how far it is from O when it does so.

1.39 ⋆⋆ A ball is thrown with initial speed v_0 up an inclined plane. The plane is inclined at an angle ϕ above the horizontal, and the ball's initial velocity is at an angle θ above the plane. Choose axes with x measured up the slope, y normal to the slope, and z across it. Write down Newton's second law using these axes and find the ball's position as a function of time. Show that the ball lands a distance $R = 2v_0^2 \sin\theta \cos(\theta + \phi)/(g \cos^2\phi)$ from its launch point. Show that for given v_0 and ϕ, the maximum possible range up the inclined plane is $R_{\max} = v_0^2/[g(1 + \sin\phi)]$. A

1.40 ⋆⋆⋆ A cannon shoots a ball at an angle θ above the horizontal ground. **(a)** Neglecting air resistance, use Newton's second law to find the ball's position as a function of time. (Use axes with x measured horizontally and y vertically.) **(b)** Let $r(t)$ denote the ball's distance from the cannon. What is the largest possible value of θ if $r(t)$ is to increase throughout the ball's flight? [*Hint:* Using your solution to part (a) you can write down r^2 as $x^2 + y^2$, and then find the condition that r^2 is always increasing.]