Formulae

Maxwell's Equations (SI units)

$$\nabla\cdot\mathbf{D} = \rho_{free} \qquad \nabla\cdot\mathbf{B}=0$$
$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \nabla\times\mathbf{H}=\mathbf{J}_{free}+\frac{\partial\mathbf{D}}{\partial t}$$
$$\mathbf{D} = \epsilon_0\mathbf{E}+\mathbf{P} \qquad \mathbf{H}=\frac{\mathbf{B}}{\mu_0}-\mathbf{M}$$

Multipole Moments

$$p^k = \int d^3r r^k\rho(\mathbf{r}) \qquad Q^{km}=\int d^3r(3r^kr^m-\delta_{km}\mathbf{r}^2)\rho(\mathbf{r}) \qquad m^k=\frac{1}{2}\int d^3r[\mathbf{r}\times\mathbf{J}(\mathbf{r})]^k$$

Laplacian in Cylindrical Coordinates

$$\nabla^2=\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial^2}{\partial z^2}$$

Laplacian in Spherical Coordinates

$$\nabla^2=\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

Legendre polynomials and Spherical Harmonics

$$\text{Rodrigues}: P_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l \qquad \text{Orthogonality}: \int_{-1}^{1}dxP_l(x)P_{l'}(x)=\frac{2}{2l+1}\delta_{ll'}$$
$$P_l^m = \frac{(-)^m}{2^l l!}(1-x^2)^{m/2}\frac{d^{l+m}}{dx^{l+m}}(x^2-1)^l$$
$$Y_{lm}(\theta,\varphi) = \sqrt{\frac{2l+1}{4\pi}}\sqrt{\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)e^{im\varphi}$$

Energy and Momentum Densities (Linear materials: $\boldsymbol{D}=\epsilon\boldsymbol{E}$, $\boldsymbol{B}=\mu\boldsymbol{H}$)

$$u = \frac{1}{2}\left(\boldsymbol{E}\cdot\boldsymbol{D}+\boldsymbol{H}\cdot\boldsymbol{B}\right), \qquad \boldsymbol{g}=\boldsymbol{D}\times\boldsymbol{B}$$

Energy Flux and Stress Tensor (Linear materials: $\boldsymbol{D}=\epsilon\boldsymbol{E}$, $\boldsymbol{B}=\mu\boldsymbol{H}$)

$$\boldsymbol{S} = \boldsymbol{E}\times\boldsymbol{H}, \qquad T^{ij}=E^iD^j+H^iB^j-\delta_{ij}u$$

Covariant Form of Maxwell Equations

$$F_{\mu\nu} = c(\partial_\mu A_\nu - \partial_\nu A_\mu) = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & cB^3 & -cB^2 \\ E_2 & -cB^3 & 0 & cB^1 \\ E_3 & cB^2 & -cB^1 & 0 \end{pmatrix}, \qquad J^\mu = (\rho c, \boldsymbol{J})$$

$$\epsilon_0 \partial_\rho F^{\mu\rho} = \frac{1}{c} J^\mu, \qquad \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0, \qquad \frac{dp_\mu}{d\tau} = \frac{q}{mc} F_{\mu\nu} p^\nu$$

Lorentz Transformation of EM Fields

$$\boldsymbol{E}'_\parallel = \boldsymbol{E}_\parallel, \qquad \boldsymbol{B}'_\parallel = \boldsymbol{B}_\parallel$$

$$\boldsymbol{E}'_\perp = \gamma(\boldsymbol{E}_\perp + \boldsymbol{v} \times \boldsymbol{B}), \qquad \boldsymbol{B}'_\perp = \gamma\left(\boldsymbol{B}_\perp - \frac{\boldsymbol{v}}{c^2} \times \boldsymbol{E}\right)$$

where the primed frame moves with velocity $\boldsymbol{v}$ relative to the unprimed frame, and $\gamma = 1/\sqrt{1 - v^2/c^2}$.

Coordinate Transformation under a Boost in the x-direction

$$x' = \gamma(x - vt), \qquad y' = y, \qquad z' = z, \qquad t' = \gamma(t - vx/c^2)$$

Green Functions

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) G = \delta(t - t')\delta(\boldsymbol{r} - \boldsymbol{r}'), \qquad G_r = \frac{\delta(t - t' - |\boldsymbol{r} - \boldsymbol{r}'|/c)}{4\pi|\boldsymbol{r} - \boldsymbol{r}'|}$$

$$\left(-\nabla^2 - k^2\right) G_k = \delta(\boldsymbol{r} - \boldsymbol{r}'), \qquad G_k = \frac{e^{ik|\boldsymbol{r} - \boldsymbol{r}'|}}{4\pi|\boldsymbol{r} - \boldsymbol{r}'|}$$

Spherical Bessel Functions

$$x \to 0: \qquad j_l(x) \sim \frac{x^l}{(2l+1)!!}, \qquad n_l(x) \sim -\frac{(2l-1)!!}{x^{l+1}}$$

$$x \to \infty: \qquad j_l \sim \frac{1}{x}\sin\left(x - \frac{l\pi}{2}\right), \qquad h_l^{(1)} \sim (-i)^{l+1}\frac{e^{ix}}{x}$$

$$j_0(x) = \frac{\sin x}{x}, \qquad n_0(x) = -\frac{\cos x}{x}, \qquad h_l^{(1,2)} = j_l \pm i n_l \tag{1}$$

Vector Spherical Harmonics

$$\boldsymbol{X}_{lm} \equiv \frac{1}{\sqrt{l(l+1)}} \boldsymbol{L} Y_{lm}, \qquad \int d\Omega \boldsymbol{X}^*_{l'm'} \cdot \boldsymbol{X}_{lm} = \delta_{ll'}\delta_{mm'}$$

$$\boldsymbol{\epsilon}_0 e^{ikz} = \frac{1}{2}\sum_{lm} i^l \sqrt{4\pi(2l+1)}\left[\frac{i}{k}\nabla \times j_l(kr) i(\epsilon_{0+}\boldsymbol{X}_{l-1} - \epsilon_{0-}\boldsymbol{X}_{l1}) + j_l(kr)(\epsilon_{0+}\boldsymbol{X}_{l-1} + \epsilon_{0-}\boldsymbol{X}_{l1})\right]$$

$$\boldsymbol{H}_0 = \frac{1}{2Z_0}\sum_{lm} i^l \sqrt{4\pi(2l+1)}\left[j_l(kr) i(\epsilon_{0+}\boldsymbol{X}_{l-1} - \epsilon_{0-}\boldsymbol{X}_{l1}) - \frac{i}{k}\nabla \times (j_l(kr)(\epsilon_{0+}\boldsymbol{X}_{l-1} + \epsilon_{0-}\boldsymbol{X}_{l1})\right]$$