Electromagnetic Theory I

Problem Set 3

Due: 23 September 2020

9. The formula for the total energy stored in a static electric field $U = (\epsilon_0/2)\int d^3x \boldsymbol{E}^2$ includes the self-energy of the charges as well as their mutual potential energy. As discussed in class the self-energy of a point charge is infinite. Study this issue for a single charge Q by spreading out a point charge in two different ways: a) Replacing the point charge by a uniformly charged spherical surface of radius δ and b) Replacing the point charge by a uniformly charged ball of radius δ. In each case, find the electric field everywhere and evaluate the integral defining U. Classical physics puts no apriori limit on how small δ could be. However, relativistic quantum mechanics implies that a particle's location cannot be known within a distance smaller than its Compton wavelength $\hbar/mc$. For cases a) and b), assume $\delta = \hbar/mc$ to get an estimate for the self energy of the electron.

10. The standard capacitor used in electrical circuits has only two conducting plates and is usually assumed to remain neutral when charged by a battery. The charge-voltage relation is then simply $Q = CV$, where Q is the charge on one of the plates (so $-Q$ is the charge on the other plate) and V is the potential difference between the plates. C is called the capacitance of the capacitor. By specializing the general two conductor situation, described by $Q_i = \sum_{j=1,2} C_{ij}V_j$, to this case, derive an expression for C in terms of the C_{ij}.

11. Consider two spherical conductors of radius R, with their centers separated by a distance $D > 2R$. The first is grounded at 0 potential, and the second is held at potential V. The problem is to find the potential everywhere outside the two spheres. One can use the method of images, but an infinite sequence of image charges inside each sphere will be required.

a)Determine the location and size of these image charges by relating the location and size of the nth charge to those of the $n-1$th charge. (Hint: Start with a charge at the center of the second conductor, with a size that brings the surface of that conductor to the desired potential V. The next image charge will be inside the first conductor of strength and location chosen so that the first conductor has $\phi = 0$, and so on.)

b)The sequence of charges inside each conductor approaches a limiting location–find the limiting location in both conductors.

c)Show that the infinite sum of potentials from the image charges converges. By keeping enough of the terms for the case $D = 4R$, compute the potential as a multiple of V, at the point midway between the two conductors on the line joining their centers, to 3 significant figures.

d) Calculate the total charge on each conductor to 3 significant figures as a multiple of $4\pi\epsilon_0 VR$ again in the case $D = 4R$.

12. J, problem 2.23.