Electromagnetic Theory I

Problem Set 8

Due: 4 November 2020

29. J, Problem 4.10

30. J, Problem 4.13

31. Consider the magnetic field produced by a current I in an infinitely long wire lying on the z axis $-\infty < z < \infty$.

a) Use symmetry arguments and Ampère's law to obtain the $\boldsymbol{B}$ field everywhere outside the wire. Express the Cartesian components of $\boldsymbol{B}$ as explicit functions of x, y, z.

b) By direct integration of each component of $\nabla \times \boldsymbol{A} = \boldsymbol{B}$, find the vector potential $\boldsymbol{A}$ for this $\boldsymbol{B}$ in Coulomb gauge, $\nabla \cdot \boldsymbol{A} = 0$.

c) Since $\nabla \times \boldsymbol{B} = 0$ "almost everywhere" we should be able to find a scalar potential such that $\boldsymbol{B} = -\nabla\phi$ "almost everywhere". By explicitly integrating the components of this equation, find a candidate for ϕ as an explicit function of x, y, z.

d) In view of the fundamental theorem of calculus

$$\phi(y) - \phi(x) \;=\; \int_x^y \boldsymbol{dl} \cdot \nabla\phi = -\int_x^y \boldsymbol{dl} \cdot \boldsymbol{B}, \tag{1}$$

explain how your result for part c) does not run afoul of Ampère's law.

32. J, Problem 5.7.