Electromagnetic Theory I

Problem Set 9

Due: 13 November 2020

33. J, Problem 5.3

34. A point magnetic dipole with moment $\boldsymbol{m} = m\hat{z}$ is placed at the center of a spherical shell with uniform magnetic permeability μ, and with inner and outer radii a, b respectively.

a) Set up the boundary equations that determine the magnetic field in the three regions $0 < r < a$, $a < r < b$, $r > b$.

b) Solve the equations of part a) to find the $\boldsymbol{B}$ and $\boldsymbol{H}$ fields in all three regions.

c) Discuss the two extreme limits $\mu \to \infty$ and $\mu \to 0$. Determine the limiting $\boldsymbol{B}$ and $\boldsymbol{H}$ fields in each region, and discuss the qualitative differences of the two limits.

d) Calculate the difference $\Delta U = U_{\mu=0} - U_{\mu=\infty}$ of the total magnetic field energy stored in the two limiting situations. Handle the divergence when $r \to 0$ by excluding the region $0 < r \leq \delta \ll a$ in the energy integral. The divergence cancels in ΔU, and you can then take $\delta \to 0$. Can you understand the sign in terms of the qualitative behavior of the fields?

35. The quantitative description of field lines is rarely discussed. In this problem we find the field line curves for the magnetic field produced by a spherical ball of radius b with permeability μ immersed in a uniform magnetic field $\boldsymbol{H}_0 = H_0\hat{z}$. From the $a \to 0$ limit of the spherical shell problem discussed in class, we know that the exterior field has the form

$$\boldsymbol{H} = H_0\hat{z} + m\frac{3z\boldsymbol{r} - r^2\hat{z}}{4\pi r^5}\,, \qquad m = \frac{4\pi b^3(\mu - \mu_0)H_0}{\mu + 2\mu_0}\,, \qquad r > b \tag{1}$$

Describing the field line curve parametrically by $\boldsymbol{r}(\lambda)$, the field line is defined by the requirement that its tangent $d\boldsymbol{r}/d\lambda$ at the point $\boldsymbol{r}(\lambda)$ be parallel to the field $\boldsymbol{H}(\boldsymbol{r}(\lambda))$ at that point. Consider the field lines of the above field in the xz plane.

a) In spherical coordinates, the curve can be specified by giving the radius as a function of the polar angle $r(\theta)$. Find the first order differential equation satisfied by $r(\theta)$.

b) Find the general solution of this equation. The values of a single integration constant will distinguish different field lines.

c) Plot a typical set of field lines for the two cases $\mu = 0$ and $\mu = \infty$. Choose them to be equally spaced in x as they approach $z = \infty$. Compare and contrast the resulting pictures in the two cases. You may restrict the field lines to the quadrant $x > 0, z > 0$.

36. J, Problem 5.22