Electromagnetic Theory I

Problem Set 10

Due: 20 November 2020

37. J, Problem 5.17. A shortcut to the solution to this problem is to relate it to an electrostaic analogue discussed in class, a charge distribution above a half space filling dielectric medium. Or you can solve it from scratch.

38. In studying a quantum particle in the presence of an electromagnetic magnetic field we used the Lagrangian

$$L = \frac{1}{2}m\dot{\boldsymbol{r}}^2 + q\dot{\boldsymbol{r}} \cdot \boldsymbol{A}(\boldsymbol{r}, t) - q\phi(\boldsymbol{r}, t) \tag{1}$$

When there is time dependence we showed near the beginning of the course that the relation between fields and potentials is

$$\boldsymbol{B}(\boldsymbol{r}, t) = \nabla \times \boldsymbol{A}(\boldsymbol{r}, t), \qquad \boldsymbol{E}(\boldsymbol{r}, t) = -\nabla\phi(\boldsymbol{r}, t) - \frac{\partial \boldsymbol{A}}{\partial t}(\boldsymbol{r}, t) \tag{2}$$

a) Show that Lagrange's equations of motion with this Lagrangian imply Newton's equation with the Lorentz force on the right side.

$$\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B} \tag{3}$$

b) Derive the Hamiltonian for this system.

c) Repeat the discussion of parts a) and b) for the relativistic Lagrangian obtained by replacing the nonrelativistic kinetic energy term in L as follows

$$\frac{1}{2}m\dot{\boldsymbol{r}}^2 \to mc^2 \left(1 - \sqrt{1 - \frac{\dot{\boldsymbol{r}}^2}{c^2}}\right) \tag{4}$$

39. J, Problem 5.25.

40. J, Problem 5.32