Electromagnetic Theory I

Problem Set 11

Due: 2 December 2020

41. Motional Emf from a Spinning Magnet (Revision of J, 6.4). Let the magnet be a uniformly magnetized sphere of radius R and magnetization $\boldsymbol{M} = M\hat{z}$, which is also a conductor with finite resistivity. Set the magnet spinning with angular speed ω about the magnetization axis. Assume the system reaches a steady state so that no current flows in the conductor's rest frame. Also assume that there is no net charge on the magnet.

a) Any charge density at rest in the magnet's rest frame must arrange itself so that the force from its electric field cancels the magnetic force from its motion in the magnet's magnetic field. Find this canceling electric field, and from it show that the steady state charge density in the bulk is $\rho = -4M\omega/3c^2 = -m\omega/\pi c^2 R^3$, where m is the total magnetic moment.

b) By examining the scalar potential for the electric field found in a) show that it is a superposition of electric monopole and quadrupole potentials. The monopole contribution must be cancelled by charge on the surface.

c) Since there is no net charge on the magnet, the field outside has no monopole contribution. From the boundary conditions at the surface (continuous tangential electric fields), derive the potential outside the sphere. Then from the difference in normal fields outside and in, show that the surface charge density is

$$\sigma(\theta) = \frac{m\omega}{3\pi c^2 R^2}\left(1 - \frac{5}{2}P_2(\cos\theta)\right) \tag{1}$$

d) By calculating the line integral of the electric field from a point on the equator to the north pole show that the induced Emf $= \mu_0 m\omega/4\pi R$.

42. J, Problem 6.5.

43. Consider the ideal circular parallel plate capacitor of radius a and plate separation $d \ll a$, hooked up to a straight current-carrying wire on the axis as pictured in the figure to J, Problem 6.14. The current in the wire varies harmonically, $I(t) = I_0 \cos\omega t = \text{Re } I_0 e^{-i\omega t}$. In this problem we neglect the effect of fringing fields, which means that the fields within the capacitor are assumed to be those between infinite parallel plates, which discontinuously drop to zero at the edge of the capacitor. This exercise will give you experience with the use of complex fields in solving physical problems.

a) In the approximation described above we may make the ansatz that the (complex) fields within the capacitor have the form

$$\boldsymbol{E} \; = \; \hat{z} f(\rho) e^{-i\omega t}, \qquad \boldsymbol{B} = \boldsymbol{\rho} \times \hat{z} g(\rho) e^{-i\omega t}$$

From the (complex) Maxwell equations determine g in terms of f and show that $f(\rho)$ satisfies the $n = 0$ Bessel equation, whose solution is $f(\rho) = A J_0(\omega\rho\sqrt{\epsilon\mu})$.

b) Now consider the low frequency (quasi-static) limit. Expand the electric and magnetic fields within the capacitor up to quadratic order in ω. Determine A up to this order by demanding charge conservation, $I_0 = -i\omega Q$, where Q is the (complex) charge on the top plate obtained by identifying the surface charge density $\sigma = \epsilon \boldsymbol{n} \cdot \boldsymbol{E}$ in terms of the electric field between the plates. (Here we are neglecting the field outside the plates.)

c) Calculate the time averaged electric and magnetic energies by integrating their time averaged densities $u_e = \text{Re}\ \epsilon \boldsymbol{E} \cdot \boldsymbol{E}^*/4$, $u_m = \text{Re}\ \boldsymbol{B} \cdot \boldsymbol{B}^*/4\mu$ over the volume between the capacitor plates.

d) From the results of part c) read off the capacitance and inductance of this system in the limit $\omega \to 0$. The resonant frequency of a series L, C alternating current circuit is $1/\sqrt{LC}$. Compare the resonant frequency, from your approximate solution to this problem, to a zero of $J_0(\omega a\sqrt{\epsilon\mu})$.

44. J, Problem 6.20