Electromagnetic Theory I

Problem Set 12

Due: 9 December 2020

45. Some derivations in the lecture notes.

a) Equation (336) of the lecture notes gives a sequence of steps that leads to the identificatioon of the momentum carried by the fields. Explain each step and discuss our identification of $\boldsymbol{g}$ with the momentum density of the fields.

b) Show that G given in Eq (348) solves Eq.(347). Note: these equation numbers are from a recent update of the lecture notes. In the previous version they were (346) and (345).

46. Plane wave solutions of Maxwell's Equations

a) Plugging the (complex) plane wave ansatz

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t}, \qquad \boldsymbol{B} = \boldsymbol{B}_0 e^{i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t} \tag{1}$$

into the sourceless vacuum Maxwell equations determine $\boldsymbol{B}_0$ and ω in terms of $\boldsymbol{E}_0$ and $\boldsymbol{k}$, and spell out any further constraints on $\boldsymbol{E}_0$. Real solutions can be constructed from these complex ones by taking their real parts. Why do we know that the real parts will be solutions?

b) Evaluate the energy and momentum densities of the real plane wave solutions (including the direction of the momentum which is a vector!). Also give the time average of these densities.

c) J, Problem 6.11 a)

d) J, Problem 6.11 b)

47. J, Problem 7.2

48. J, Problem 7.3, but only for the case of polarization perpendicular to the plane of incidence.