Electromagnetic Theory II

Problem Set 1

Due: 20 January 2021

1. J, Problem 11.4.

2. J, Problem 11.7. In this problem the starting shots can be thought of as two events in spacetime. You are asked to find a Lorentz frame for which, depending on the relationship of T to d/c, these events either occur at the same time at different points in space or at the same point in space at different times. *Hint*: consider Lorentz boosts in the y direction, to find these frames. Add a third part

c) In frame K the trajectories of the two sprinters can be taken as $(x_1(t), y_1(t)) = (v_1 t, 0, 0)$ and $(x_2(t), y_2(t)) = (v_2(t-T), d, 0)$. Transform these trajectories to the K' frame in both cases. That is work out $(x'_{1,2}(t'), y'_{1,2}(t'))$.

3. One way to compare length measurements in different frames is to compare how a wave packet looks in different frames. Let the function $f(x)$, independent of y, z, describe a symmetric profile in the x coordinate with a definite width w. Then $\psi(x,t) = f(x - ct)$ solves the wave equation

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi = 0 \tag{1}$$

in the unprimed frame.

a) Find the function $\psi'(x', t')$ that describes how this packet looks in the primed frame which is moving in the negative x direction at velocity v, and show that it solves the wave equation in the primed system.

b) What is the width w' of this profile as determined in the primed system? Notice that w'/w is *not* the usual Lorentz contraction factor.

c) To understand this difference, consider a rod of rest length L_0 parallel to the x-axis, moving in the positive x direction with velocity u in the unprimed frame. What is its velocity in the primed frame?

d) Calculate the lengths L, L' of the rod as seen in the unprimed and primed systems respectively. Compare L'/L to w'/w, and comment.

4. We have represented a general Lorentz transformation as a 4×4 matrix $\Lambda^\mu{}_\nu$. An infinitesimal Lorentz transformation can be written $\Lambda^\mu{}_\nu = \delta^\mu{}_\nu + \epsilon^\mu{}_\nu$ where the $\epsilon^\mu{}_\nu \ll 1$.

a) Show that the restriction on Λ required of Lorentz transformations is equivalent to the statement that the matrix $\epsilon_{\mu\nu} = \eta_{\mu\rho}\epsilon^{\rho}{}_{\nu}$ is antisymmetric $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu}$. In particular, this means $\epsilon^{0}{}_{i} = +\epsilon^{i}{}_{0}$, $i = 1, 2, 3$ and $\epsilon^{i}{}_{j} = -\epsilon^{j}{}_{i}$, $i, j = 1, 2, 3$.

b) For an infinitesimal boost in the x direction the matrix ϵ^{μ}_{ν} is therefore

$$\epsilon \quad \equiv \lambda K_x = \lambda \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2}$$

By explicitly evaluating the expansion

$$\Lambda = e^{\lambda K_x} = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} K_x^n$$

when λ is *not* small, show that the matrix Λ gives a finite boost in the x-direction, and identify the boost velocity.

c) Similarly show that a finite rotation about the z-axis is the exponential of an infinitesimal rotation.