Electromagnetic Theory II

Problem Set 6

Due: 24 February 2021

21. J, Problem 7.4.

22. J, Problem 7.13.

23. In our class discussion of plane waves in a plasma in the presence of a uniform magnetic field $\boldsymbol{B}_0$, we derived a formula for the displacement field $\boldsymbol{D}$ in terms of the electric field:

$$\boldsymbol{D} \;=\; \epsilon_0 \boldsymbol{E} + \boldsymbol{P} = \epsilon_0 \left[\boldsymbol{E} - \frac{\omega_p^2}{\omega^2 - \omega_B^2} \left(\boldsymbol{E} - i\boldsymbol{E} \times \frac{\boldsymbol{\omega}_B}{\omega} - \frac{\boldsymbol{\omega}_B}{\omega} \frac{\boldsymbol{\omega}_B}{\omega} \cdot \boldsymbol{E} \right) \right] \tag{1}$$

where $\boldsymbol{\omega}_B = e\boldsymbol{B}_0/m$ is the cyclotron frequency for the magnetic field. We can express this relation in terms of a matrix dielectric constant $D^k = \epsilon_{kl} E^l$. Find the three eigenvalues and their eigenvectors of the matrix ϵ_{kl}. Relate your results to the propagation of a plane wave in the plasma.

24. To understand the behavior of a wave packet,

$$\psi(\boldsymbol{r}, t) = \int d^3k f(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega(\boldsymbol{k})t}, \tag{2}$$

it is useful to have an example where all the integrals can be done explicitly for all t. Consider a Gaussian form $f(\boldsymbol{k}) = Ae^{-(\boldsymbol{k}-\boldsymbol{k}_0)^2/(2\delta^2)}$. Assuming a quadratic behavior $\omega = \boldsymbol{k}^2/2\alpha$ for the frequency renders the integrand a Gaussian for all t.

a) Calculate the integral that defines ψ in this case, putting the answer in a form $K(t)e^{i\boldsymbol{k}_0\cdot\boldsymbol{r} - (1 - it\delta^2/\alpha)(\boldsymbol{r} - \boldsymbol{r}_0(t))^2/\Delta^2(t)}$ and find explicit expressions for $K(t), \Delta(t), \boldsymbol{r}_0(t)$.

b) Show that $d\boldsymbol{r}_0/dt = \boldsymbol{v}_g$ is just the group velocity we have introduced in describing the motion of wave packets with sharply peaked wave number distributions.

c) Discuss how the wave function spreads with time. Find the time it takes to double in width. What are the restrictions on an initial wave packet for which there is negligible spreading throughout a scattering process in which there is a distance L from production location of the incoming beam to the location of the detector.?