Electromagnetic Theory II

Problem Set 12

Due: 14 April 2021

44. J, Problem 10.3. The absorption cross section asked for in part b) is the power absorbed by the scatterer divided by the incident flux. For large conductivity the power absorbed per unit area is $|\boldsymbol{H}_\parallel|^2/2\sigma\delta$ given by J, Eq.(8.12) or Eq. (606) in Chapter 10 0f our notes. Here $\boldsymbol{H}_\parallel$ is the tangential magnetic field at the surface of the conductor, which you find in part a).

45. In our class discussion of scattering of electromagnetic waves from a perfectly conducting sphere we showed that the long wavelength limit of the exact scattering amplitude gave the behavior

$$\boldsymbol{f} \sim -\sqrt{\frac{\pi}{3}} k^2 R^3 \left[(\epsilon_{0+} \boldsymbol{X}_{1-1} + \epsilon_{0-} \boldsymbol{X}_{11}) + (2i)\hat{k} \times (\epsilon_{0+} \boldsymbol{X}_{1-1} - \epsilon_{0-} \boldsymbol{X}_{11}) \right]. \tag{1}$$

Using the explicit forms for the vector spherical harmonics, show that this reproduces the forms

$$\begin{aligned} \boldsymbol{f} &= -\frac{k^2 R^3}{2} \left[-\hat{k} \times (\hat{k}_0 \times \boldsymbol{\varepsilon}_0) + 2\hat{k} \times (\hat{k} \times \boldsymbol{\varepsilon}_0) \right] \\ \boldsymbol{\varepsilon}^* \cdot \boldsymbol{f} &= k^2 R^3 \left[-\frac{1}{2} (\hat{k} \times \boldsymbol{\varepsilon}^*) \cdot (\hat{k}_0 \times \boldsymbol{\varepsilon}_0) + \boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0 \right] \end{aligned} \tag{2}$$

obtained in our earlier discussion of the long wavelength approximation.

46. J, Problem 10.7

47. J, Problem 10.9. The lowest order approximation asked for in part a) is simply the Born approximation. The integral over angles to form the total cross section in part b) is best done by changing variables from θ to q, where $q^2 = (\boldsymbol{k} - k\hat{z})^2 = 2k^2(1 - \cos\theta)$. Since the result of part a) is simply the $ka \to \infty$ limit of that in part b), it suffices to only do the calculation in part b).