

String Theory

Problem Set 1

Due: 25 January 2008

1. Consider all the diagrams of ϕ^3 theory, with coupling g and mass m , that contribute to the 6 point tree scattering amplitude.
 - a) Characterize them by drawing a graph with each distinct allowed topology and stating the number of distinct diagrams that contribute to each.
 - b) Then write down the complete expression for the value of a typical diagram for each topology—specifying all the kinematic invariants in detail.
 - c) Describe how the rest of the diagrams can be systematically obtained from the ones you give by substitution rules.
2. Consider a scalar field theory with a light field ϕ , mass m and a heavy field Φ , mass $M > 2m$. Assume there is an interaction term $G\phi^2\Phi$ in the Lagrangian.
 - a) Calculate the self energy diagram for Φ .
 - b) By evaluating its imaginary part, find the width of the resonance coupling to Φ and confirm that it is positive (for real G).
 - c) Calculate the tree level decay rate for the unstable Φ and compare it to the width.
3. Look up, in the particle data book, the mass and angular momentum data for the $Q\bar{Q}$ mesons made up of the “light” quarks, up, down, and strange. Group the mesons according to their “flavor” quantum numbers, charge conjugation, and relative parity. For each group plot the data on a J/M^2 graph. Assess the extent to which the points fall on linear Regge trajectories, $J(M^2) \approx \alpha' M^2 + \alpha_0$, and present a fit to the parameters α', α_0 . Is there evidence that either of these parameters is universal?
4. For the mesons tabulated in problem 3, tabulate the ratios Γ_{total}/M , and assess the viability of a “narrow resonance approximation”.