String Theory

Problem Set 2

Due: 18 February 2008

1. As a good exercise on the operator formalism, start with the operator matrix element for the completely factorized n-point function in multiperipheral configuration

$$\langle 0|e^{ip_n\cdot q_0}V_{p_{n-1}}B(L_0-\alpha_0,\alpha_0)V_{p_{n-2}}B\cdots V_{p_2}e^{ip_1\cdot q_0}|0\rangle \tag{1}$$

and derive the Koba-Nielsen form of the amplitude's integral representation. This is essentially reversing the logic we followed in class.

2. Prove the following identity:

$$\sum_{k=1}^{n-1}k^2=\frac{n(n-1)(2n-1)}{6} \tag{2}$$

3. The vertex operator $V(z)=z^{L_0}V_pz^{-L_0}$ satisfies the following property

$$[L_n,V(z)]=z^{n+1}\frac{dV(z)}{dz}+n\alpha_0z^nV(z) \tag{3}$$

which we interpreted (at $\alpha_0=0$) as the change of V under the infinitesimal transformation $z\to z+\alpha z^{n+1}$. Using this identity work out the corresponding finite transformation $e^{\xi L_n}V(z)e^{-\xi L_n}=V(f(z))$, and find $f(z)$. Note that for $n=\pm 1,0$ these are just various projective transformations. As an optional second part consider the finite transformation in the case $\alpha_0\neq 0$.

4. An exercise in gauge field theory: Calculate the tree approximation to the on-shell four gluon scattering amplitude in pure Yang-Mills theory, $\mathcal{L}=-\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}/4$, with $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu-ig[A_\mu,A_\nu]$, for general polarizations for the four gauge particles.