String Theory

Problem Set 4

Due: 24 March 2008

1. Verify the super-Virasoro algebra in the Neveu-Schwarz sector of the spinning string model:

$$
\begin{aligned}
[L_n, a_m^\mu] &= -m a_{m+n}^\mu, \qquad [L_n, b_r^\mu] = -(r+n/2) b_{r+n} && (1)\\
[G_r, a_m^\mu] &= -m b_{r+m}^\mu, \qquad \{G_r, b_s^\mu\} = a_{r+s}^\mu && (2)\\
[L_n, L_m] &= (n-m) L_{m+n} + \frac{D}{8}(n^3 - n)\delta_{n,-m} && (3)\\
[L_n, G_r] &= (n/2 - r) G_{r+n} && (4)\\
\{G_r, G_s\} &= 2L_{r+s} + \frac{D}{2}(r^2 - 1/4)\delta_{r,-s} && (5)
\end{aligned}
$$

2. The vertex operator for the emission of the *Picture 2* "gluon" state in the Neveu-Schwarz model, $\epsilon(k)\cdot b_{-1/2} e^{ik\cdot q_0}|0\rangle$, can be obtained from

$$V(k,\epsilon) \equiv \{G_r, : \epsilon(k)\cdot H(z) e^{ik\cdot x(z)} :\} \qquad (6)$$

   (a) Perform the anti-commutator to obtain the vertex operator in terms of $H^\mu(z)$, $\mathcal{P}^\mu(z)$, and $: e^{ik\cdot x(z)} :$.

   (b) Using this expression in the *Picture 2* formula for the amplitude, calculate the 4 "gluon" scattering amplitude

$$\langle 0|\epsilon_4\cdot b_{1/2} e^{ik_4\cdot q_0} V(k_3,\epsilon_3) \frac{1}{L_0 - 1/2} V(k_2,\epsilon_2)\epsilon_1\cdot b_{-1/2} e^{ik_1\cdot q_0}|0\rangle. \qquad (7)$$

   (c) Compare your answer to the 4 gluon calculation in Yang-Mills theory that you did in problem 4 of set 2, and to the 4 "gluon" amplitude you obtained in problem 1 of set 3.

3. The proof of the no-ghost theorem in the various string models uses a basis of states generated by the super-Virasoro generators and and $k\cdot b_r$, $k\cdot a_n$ for some light-like k^μ. For states of mode number $N = 3/2$ in the Neveu-Schwarz model this basis consists of 6 elements:

$$
\begin{aligned}
|1\rangle &= G_{-1/2} L_{-1}|T\rangle, \quad |2\rangle = G_{-3/2}|T\rangle, \quad |3\rangle = L_{-1} k\cdot b_{-1/2}|T\rangle && (8)\\
|4\rangle &= G_{-1/2} k\cdot a_{-1}|T\rangle, \quad |5\rangle = k\cdot b_{-3/2}|T\rangle, \quad |6\rangle = k\cdot b_{-1/2} k\cdot a_{-1}|T\rangle && (9)
\end{aligned}
$$

   where the ket $|T\rangle$ satisfies $(G_r, L_n, k\cdot b_r, k\cdot a_n)|T\rangle = 0$ for $r, n > 0$, and $L_0|T\rangle = \lambda|T\rangle$. Calculate all 36 elements of the matrix $M_{ij} = \langle i|j\rangle$, and confirm that it is triangular.