String Theory

Problem Set 6/ Final Project

Due: 30 April 2008

- Derive, based on the tree theorem with full explanation and all details, the planar open string loop diagram for the Neveu-Schwarz model. Assume an even number of tachyons, N, coming into the diagram.
- Transform to cylinder variables and discuss the divergences associated with the region $q \sim 0$, assuming the critical dimension $D = 10$.

$$
\begin{aligned}
\mathcal{M}_{\substack{\text{Planar}\\ \text{Loop}}} = & \left(\frac{1}{8\pi^2\alpha'}\right)^{D/2} \int \prod_{k=2}^{N} d\theta_k \int_0^1 \frac{dq}{q} q^{-(D-2)/8} w^{(D-10)/16} \frac{\prod_r (1+q^{2r})^{D-2}}{\prod_n (1-q^{2n})^{D-2}} \\
& \prod_{l<m} \left[\sin\frac{\theta_{lm}}{2} \prod_n \frac{(1-q^{2n}e^{i\theta_{lm}})(1-q^{2n}e^{-i\theta_{lm}})}{(1-q^{2n})^2} \right]^{2\alpha' k_l \cdot k_m} \\
& \langle k_1 \cdot H(\theta_1) k_2 \cdot H(\theta_2) \cdots k_N \cdot H(\theta_N) \rangle
\end{aligned}
$$

where the average over H fields is evaluated via Wick's theorem for free fermi fields with contractions:

$$
\langle k \cdot H(\theta+\delta) k' \cdot H(\theta) \rangle = k \cdot k' \left(\sin\frac{\delta}{2}\right)^{-1} \prod_n \frac{(1-q^n)^2(1-q^{2n})^2(1+q^{2n-1}e^{i\delta})(1+q^{2n-1}e^{-i\delta})}{(1-q^{2n}e^{i\delta})(1-q^{2n}e^{-i\delta})}
$$

and the range of integration is

$$
0 = \theta_1 < \theta_2 < \cdots < \theta_N < 2\pi
$$

The index r ranges over positive half odd integers, the index n over positive integers, and $l, m \in [1, \cdots, N]$