Example Showing Unending Inflation Compatible with the Two-loop Woodard-Tsamis Calculation

Define the function

\[
C_3(x) = \frac{1}{3} \left( e^x + 2e^{-x/2} \cos(x \sqrt{3}/2) \right) \\
C_3(x) = \frac{1}{3} \left( \exp(x) + \exp(x e^{2\pi i/3}) + \exp(x e^{-2\pi i/3}) \right) \\
= \sum_{k=0}^{\infty} \frac{1}{(3k)!} x^{3k} = 1 + \frac{1}{3!}x^3 + \ldots
\]

This function has exponential growing behavior at large \( x > 0 \) and oscillatory falling exponential behavior at large \( -x > 0 \). There are zeroes on the negative \( x \) axis, the least negative being at approximately \( x \approx -1.84981 \).

Then the function

\[
a(t) = \frac{e^{H_0 t}}{C_3(\xi(G\Lambda)^{2/3}H_0 t)} \sim e^{H_0 t} \left( 1 - \frac{1}{3!} \xi^3 (G\Lambda)^2 (H_0 t)^3 + O((G\Lambda)^4(H_0 t)^6) \right) \\
\frac{\dot{a}}{a} \sim H_0 \left( 1 - \frac{\xi^3}{2} (G\Lambda)^2 (H_0 t)^2 + O((G\Lambda)^4(H_0 t)^5) \right)
\]

shows the two loop small \( G \) behavior of Tsamis and Woodard without an end to inflation. There is simply a renormalization of the Hubble constant

\[
H_{eff} = H_0(1 - \xi(G\Lambda)^{2/3})
\]

This particular ansatz for the large time behavior has a singularity in the past however.