Example Showing Unending Inflation Compatible with the Two-loop Woodard-Tsamis Calculation

Define the function

$$\begin{aligned} C_3(x) &= \frac{1}{3}\left(e^x + 2e^{-x/2}\cos(x\sqrt{3}/2)\right) \\ C_3(x) &= \frac{1}{3}\left(\exp(x) + \exp(xe^{2\pi i/3}) + \exp(xe^{-2\pi i/3})\right) \\ &= \sum_{k=0}^{\infty}\frac{1}{(3k)!}x^{3k} = 1 + \frac{1}{3!}x^3 + \cdots \end{aligned} \tag{1}$$

This function has exponential growing behavior at large $x > 0$ and oscillatory falling exponential behavior at large $-x > 0$. There are zeroes on the negative x axis, the least negative being at approximately $x \approx -1.84981$.

Then the function

$$\begin{aligned} a(t) &= \frac{e^{H_0 t}}{C_3(\xi(G\Lambda)^{2/3}H_0 t)} \sim e^{H_0 t}\left(1 - \frac{1}{3!}\xi^3(G\Lambda)^2(H_0 t)^3 + O((G\Lambda)^4(H_0 t)^6\right) \\ \frac{\dot{a}}{a} &\sim H_0\left(1 - \frac{\xi^3}{2}(G\Lambda)^2(H_0 t)^2 + O((G\Lambda)^4(H_0 t)^5\right) \end{aligned} \tag{2}$$

shows the two loop small G behavior of Tsamis and Woodard without an end to inflation. There is simply a renormalization of the Hubble constant

$$H_{eff} = H_0(1 - \xi(G\Lambda)^{2/3}) \tag{3}$$

This particular ansatz for the large time behavior has a singularity in the past however.