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The Language of String[1]

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Abstract

I summarize the talk I gave at the Pierrefest celebration of Pierre Ramond's sixtieth birthday. (Gainesville, Florida, 1 February 2003)

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1 Introduction

All known physics ($E \leq 1\mathrm{TeV}$) can be described[3] with exquisite precision by quantum field theory.

That statement, true in the 1970's and still true today, seems to kill string theory, at least for its original purpose of explaining the physics of hadrons ($E \sim$0.1-10GeV). One way to rescue string theory from oblivion is always Joel Scherk's observation that, once divorced from the physics of strong interactions, the string tension T_0 could be assumed arbitrarily high, beyond the reach of current experiments. Then string theory enters the realm of speculation about what physics we may find in future experiments. Initially, because string seemed promising for quantum gravity, the guess was put at $T_0 \sim M^2_{\text{Planck}}$ well beyond *any* future experiments. But there is more recent speculation that it might be as low as $10\mathrm{TeV}^2$, just around the LHC corner.

But in recent years, especially with the advent of Maldacena's (String on AdS)/(SUSY Gauge Theory) duality, the old idea that string might provide an alternate language for ordinary quantum field theory, has been revived. Indeed, I believe it should be possible to give a string description of QCD. This would bring string full circle back to its roots, where it can shed light on some still unanswered questions, *e.g.* confinement, in strong interaction physics. I will tell you a little about my recent work, initiated last year with Korkut Bardakci, along these lines in the second half of my talk. But let me first bring Pierre Ramond, whose career we are celebrating today, into the story.

This is easy to do by simply recounting a little history about how the language of string evolved. For the purposes of today's talk, I will understand a string description to be simply one based on worldsheet dynamics. This language for dual models was almost entirely developed within a year or two following the appearance of Veneziano's four meson dual resonance amplitude in 1968.

I started graduate studies at Berkeley in September 1968, not long after Veneziano's paper, but I didn't participate in the stunning series of steps that led from Veneziano's innocent looking 4 point function to a full-fledged perturbative dual resonance (string) theory that was as compelling as the usual Feynman diagram expansion of quantum field theory[4]. It differed from QFT by the ubiquitous presence of a scale of nonlocality $\alpha'(= 1/2\pi T_0)$. By the time I started my thesis research in 1969, with Stanley Mandelstam as my adviser, the underlying worldsheet description of these dual models was being uncovered by Nambu, Nielsen, and Susskind. Soon thereafter Nambu and Goto proposed the worldsheet action for string. Among other things, Nambu's work showed that Virasoro's Ward-like identities $L_n = 0$, necessary to remove ghosts (negative probability processes), were consequences of worldsheet reparametrization invariance.

Briefly, the dynamical variables of the N-G action are string coordinates $x^\mu(\sigma,\tau)$ and their canonical conjugates $\mathcal{P}^\mu$, which are regarded as quantum fields on the two-dimensional worldsheet paramterized by σ, τ. It is very convenient to parametrize the worldsheet with

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[3] Modulo one or two technical difficulties like prediction of the hadron mass spectrum and the non-renormalizability of quantum gravity

[4] This was the work of many people: Bardakci, Ruegg, Chan, Goebel, Sakita, Virasoro, Mandletam, Fubini, Veneziano, Koba, and Nielsen to name but a few.

light-cone coordinates: $x^+ = \tau$ and $\mathcal{P}^+ = 1$, in which case the phase space worldsheet action is

$$S = \int d\tau \int_0^{p^+} d\sigma \quad \left(\dot{\boldsymbol{x}} \cdot \boldsymbol{\mathcal{P}} - \frac{1}{2}\boldsymbol{\mathcal{P}}^2 - \frac{T_0^2}{2}\boldsymbol{x}'^2 \right). \tag{1}$$

I spent my first year of research engaged in technical issues such as calculating loop amplitudes and clarifying Virasoro's L_n algebra. But by the end of 1970 I was very eager to make a more substantive contribution to particle physics. The more senior physicists working at that time on dual models at Berkeley (Bardakci, Halpern, Mandelstam, Virasoro) pretty much concurred that there were two big problems left untouched by the developments to date: the first of these was finding dual amplitudes for fermions. Bardakci and Halpern were already attacking this problem by adjoining spinor valued worldsheet fields $\psi_\alpha(\sigma,\tau)$ to the coordinates $x^\mu(\sigma,\tau)$[5]. Unfortunately, the ghost (negative probability) problems faced here were much more severe than in bosonic models, and to get around them Bardakci and Halpern were led to consider quite complicated interaction schemes. This whole program seemed so far over my head that I felt I had no hope of contributing to it.

2 Reminiscences on the Ramond Sector

But then came the much simpler proposal of Pierre Ramond (1970-71). His line of thought was an elegant generalization of the Dirac equation:

$$\gamma \cdot p\psi = 0 \to [\Gamma_1(\sigma) \cdot \mathcal{P}(\sigma) + T_0\Gamma_2(\sigma) \cdot x'(\sigma)]\Psi = 0. \tag{2}$$

Notice that apart from the capitalization of certain symbols, the equation on the right is really an infinite family of equations labelled by the worldsheet coordinate σ. Reading this paper was my first intellectual encounter with Pierre and his idea struck a chord in me. I was confident I could do something with his program. When I asked my senior colleagues at Berkeley why Ramond had not already solved the fermion problem in dual models, they were more or less unanimous: his was an interesting proposal for free fermions, but there was not yet a viable scattering amplitude for fermions. This judgment may seem a bit harsh, but remember that at Berkeley in the sixties, if you didn't have an S-matrix, you had nothing! So I made finding a scattering amplitude for Pierre's fermions my first research challenge.

Let me stress that writing down candidate dual amplitudes which included Pierre's fermions was pretty easy: the problem was making sure that there were no negative probabilities (ghosts). For the bosonic amplitudes this was achieved through Virasoro's infinite family of "Ward-like Identities" L_n. Theories with fermions needed those plus an equal number of new fermionic ones F_n, which Pierre proposed in his paper. Together L_n, F_n formed what we now call a super-conformal algebra. The challenge was to find scattering amplitudes which respected the full superalgebra of Ward Identities. I didn't make much progress until André Neveu came to Berkeley in April 1971 to talk about his model with John Schwarz,

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[5] this idea was later exploited by Green and Schwarz in an alternate formulation of superstring theiry.

which presented amplitudes for bosons only that nonetheless apparently respected a similar super-algebra set of Ward-identities (called L_n, G_r). At the end of André's seminar, there was an implicit challenge: although they had accumulated, by brute force level by level calculations, convincing evidence that the Ward-identities were true, they didn't really understand how they worked and had not yet been able to prove them.

Because I had already been thinking along these lines, and André's challenge inspired me to quickly discover two things. Firstly, I found a nice proof of all the Ward-identities for the Neveu-Schwarz model, and secondly, used the methods of that proof to construct fermion scattering amplitudes which had the same property. The basic identity was

$$V_\pi = k \cdot HV_0 = \{G_r, V_0\} \rightarrow G_r, V_\pi = [G_r^2, V_0] = [L_{2r}, V_0]. \tag{3}$$

This identity could be used to derive a new picture for the Neveu-Schwarz model, and in the new picture they could be used to derive the Ward identities. For fermions I just had to substitute $G_r \rightarrow F_n$, and I had met my first self-imposed research challenge!

After a period of euphoria over this achievement, it dawned on me how much of my result was due to Pierre's elegant proposal, and how small a step I had actually taken to make the scattering amplitude. I still like to think of this small piece of work as a collaboration (albeit a virtual one) with Pierre. I called him up at NAL and suggested that we write up the work together. But he seemed quite taken aback by my suggestion, and he quite generously said the work was mine so I should write it alone, which is what happened.

Pierre's solution of the fermion problem (as well as that of Bardakci and Halpern and earlier work of Clavelli and Ramond) was among the first steps to enlarge the worldsheet dynamical system beyond the string coordinates x^μ. One could introduce other variables into the worldsheet dynamics in addition to coordinates and spin, thus widening the scope of the formalism. We shall see that this flexibility allows the worldsheet formalism to be applied even in theories where an actual physical string state doesn't exist.

3 The Second Big Problem with String

If dual models had failed to describe fermions, string theory would be dead in the water as a physical theory, because nature most assuredly contains fermions. The second big shortfall of string, though, seems equally devastating. Contemporaneously with the development of string just described, far-reaching experiments were being conducted and interpreted at SLAC. These experiments convincingly demonstrated that the constituents of the hadron were point partons (quarks, gluons) much like the constituents of an atom. Such an internal structure has an effortless explanation in the context of quantum field theory, which after all is a theory of interacting point particles. In contrast, the physics of string shows no trace of such constituents. This seemed to kill string theory as a theory of hadrons. One could of course keep string theory alive by following Joel Scherk's suggestion and postponing its relevance to higher energy scales and later experiments. But there is another way.

Instead I want to turn now to the possibility that string is a new way of describing quantum field theory. As such, string might be a valuable tool to better understand quark confinement, bringing string full circle back to it roots. Actually, this idea goes back to the very early days of string. In 1970 Sakita and Virasoro, and independently, Nielsen and Olesen proposed that string propagation should be interpreted as an approximation to the sum of the large planar Feynman diagrams of some field theory. They called these diagrams fishnets. The nature of such an approximation was obscure however. One might justify large diagrams by a strong coupling limit, but how could it make any sense to consider only the planar diagrams? In 1974, 't Hooft solved this latter problem by showing that in SU(N_c) gauge theory the limit $N_c \to \infty$, with $\lambda = \alpha_s N_c/\pi$ fixed, singles out precisely the planar diagrams. Then the planar approximation to QCD would rely on the proposition that $3 >> 1$. Note however that small planar diagrams are just as important as large ones for $\lambda < 1$. To get a fishnet approximation requires the further parametric limit $\lambda \gg 1$. Note that 't Hooft's observation provides very important information: the existence of a parametric limit that gives planar diagrams assures that a planar diagram approximation is self-consistent.

4 Fishnets

In 1977, after several years devoted to the MIT Bag model, I decided to return to string theory and try to make the ideas sketched above systematic by transcribing everything to light-cone quantization. Light-cone coordinates are defined as $x^\pm = (x^0 \pm x^3)/\sqrt{2}$. Then x^+ is the quantum evolution parameter, and the Hamiltonian conjugate to this time is p^-.

A propagator in a Feynman diagram takes the form.

$$\Delta(\mathbf{p}, p^+, x^+) = \frac{\theta(x^+)}{2p^+} e^{-ix^+\mathbf{p}^2/2p^+} \to \frac{\theta(\tau)}{2p^+} e^{-\tau\mathbf{p}^2/2p^+}, \tag{4}$$

where we take $p^+ > 0$. If we only have cubic vertices a fishnet diagram is of the type shown in Fig. 1. To formulate a strong coupling limit to justify the focus on large diagrams, I discretized $\tau = ka$ and $p^+ = lm$, with $k, l = 1, 2, 3, \ldots$. Then $\lambda \to \infty$ implies that the

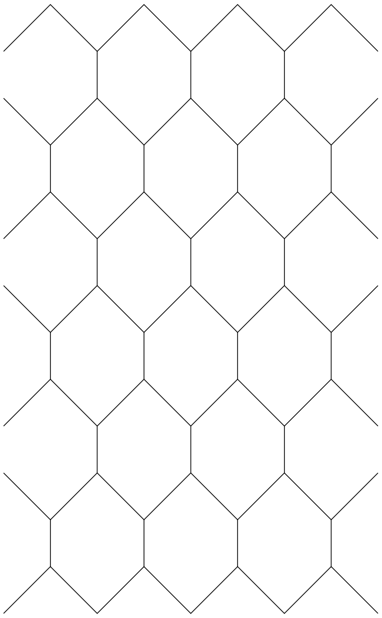


Figure 1: A fishnet diagram

dominant graphs have $k = 1$ and $l = 1, 2$. In this case the fishnet diagrams just go over to the free string propagator, with $T_0 \propto m/a$.

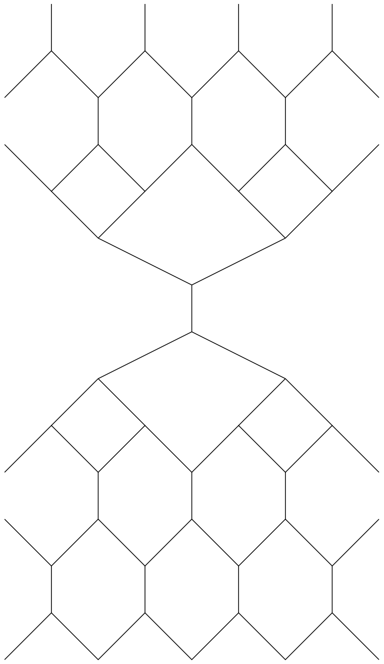


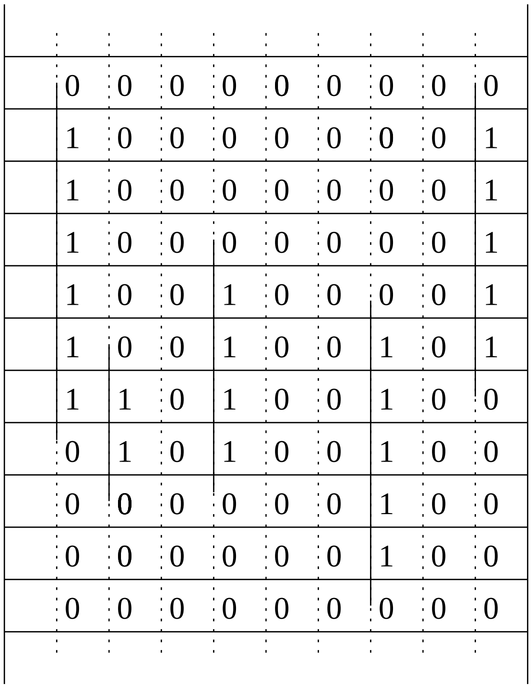
Figure 2: A finite momentum parton in a fishnet diagram.

5 Where are the partons?

Evidently, strong coupling forces all constituents to have zero momentum. The diagram in Fig. 2 shows a fluctuation which produces a finite momentum constituent. It is clearly suppressed by many powers of $1/\lambda$. On the other hand once formed such a constituent should be long lived because of asymptotic freedom, and experiment confirms that they play a prominant role in real hadrons. So to be realistic even in the planar approximation we have to work with $\lambda \leq 1$, and twenty five years ago it seemed hopeless to even formulate the planar sum in that regime, let alone solve it.

6 AdS/CFT

My interest in returning to attack this problem was revived by the successes of the AdS/CFT correspondence. In that context there were very powerful arguments that a string description of $\mathcal{N} = 4$ supersymmetric gauge theory was indeed feasible. I strongly believed that a string theory/field theory equivalence should be more universal.



0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	1
1	0	0	1	0	0	0	0	1
1	0	0	1	0	0	1	0	1
1	1	0	1	0	0	1	0	0
0	1	0	1	0	0	1	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0

Figure 3: Worldsheet representation of a low order planar Feynman diagram.

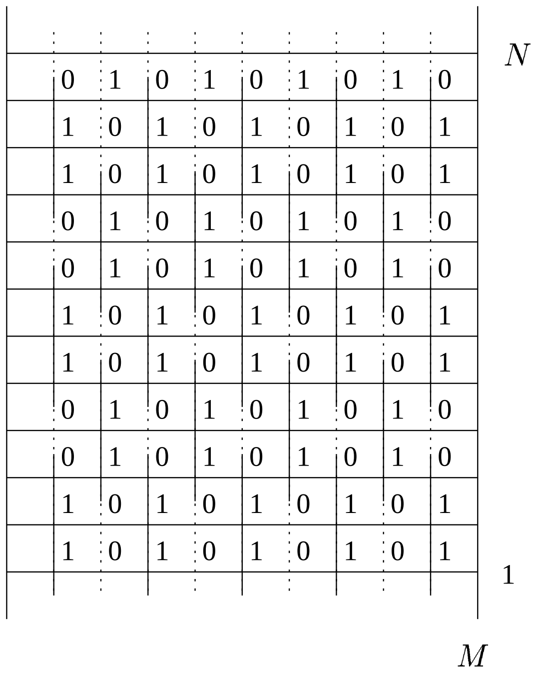
7 BT formalism

In fall 2001 I was on sabbatical at Berkeley and found that Korkut Bardakci was also interested in returning to fishnet ideas so we worked together on the problem. I think we made significant progress, but here I only have time to try to explain our breakthrough for dealing with finite momentum partons in the context of fishnets. It is based on the identity

$$\exp\left\{-\frac{\tau}{2p^+}(\mathbf{q}_M-\mathbf{q}_0)^2\right\} = \int DcDbD\mathbf{q}\; e^{-S_0} \tag{5}$$

$$S_0 = \int_0^\tau d\tau \int_0^{p^+} d\sigma \left(b'c' - \frac{1}{2}\mathbf{q}'^2\right), \tag{6}$$

which expresses the propagator for a finite momentum field quantum in terms of the propagation of string bits each carrying infinitesimal momentum. A planar Feynman diagram where propagators use this representation is shown in Fig. 3. In this picture I have indicated how to keep track of the structure of the diagram by coupling the worldsheet field $\boldsymbol{q}$ to an Ising-like two state system, with the states labelled by $0, 1$. Thus the sum of planar diagrams can be represented as the worldsheet dynamics of a non-interacting string in a nontrivial backfground represented by the Ising spins. A fishnet diagram is represented in this language in Fig. 4. Note the antiferromagnetic arrangement of Ising spins.



0	1	0	1	0	1	0	1	0
1	0	1	0	1	0	1	0	1
1	0	1	0	1	0	1	0	1
0	1	0	1	0	1	0	1	0
0	1	0	1	0	1	0	1	0
1	0	1	0	1	0	1	0	1
1	0	1	0	1	0	1	0	1
0	1	0	1	0	1	0	1	0
0	1	0	1	0	1	0	1	0
1	0	1	0	1	0	1	0	1
1	0	1	0	1	0	1	0	1

Figure 4: Worldsheet representation of a fishnet, where $T = Na, p^+ = Mm$.

8 Conclusions

- String theory may describe new physics and resolve problems with quantum gravity.
- But less speculatively, it may provide a powerful tool to analyze unsolved problems of old physics.
- Let's hope that it will do both!