Quantum Field Theory

Problem Set 10

Due: 29 March 2019

Reading: Lecture Notes, Chapters 6-7.

36 and 37. We shall soon find, when we complete the quantization of the electromagnetic field, that the photon propagator can be taken to be

$$\langle 0|TA_\mu(x)A_\nu(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{-i\eta_{\mu\nu}e^{i(x-y)\cdot p}}{p^2-i\epsilon}.$$

Anticipatiing this result, the Feynman amplitudes for elastic e^+e^- and e^-e^- scattering are

$$\begin{aligned}\mathcal{M}_{e^+e^-} &= -ie^2\left[\frac{\bar{v}(q)\gamma_\mu v(q')\bar{u}(p')\gamma^\mu u(p)}{t} - \frac{\bar{u}(p')\gamma_\mu v(q')\bar{v}(q)\gamma^\mu u(p)}{s}\right]\\ \mathcal{M}_{e^-e^-} &= ie^2\left[\frac{\bar{u}(q')\gamma_\mu u(q)\bar{u}(p')\gamma^\mu u(p)}{t} - \frac{\bar{u}(p')\gamma_\mu u(q)\bar{v}(q')\gamma^\mu u(p)}{u}\right]\end{aligned} \tag{1}$$

where $s = -(q+p)^2$, $t = -(q'-q)^2$ and $u = -(p'-q)^2$. Draw the Feynman diagrams depicting the two terms on the right side. Then calculate, to lowest order, the differential cross-section for

36. e^+e^- scattering (Bhabha scattering) and

37. e^-e^- scattering (Moller scattering)

for the case that both particles in the initial state are unpolarized and the final spins are unobserved. Notice that the expressions for the squares of the amplitudes summed over spins for these two processes are simply related to each other by the substiturions $q \leftrightarrow -q'$. and interchanging some of the 4-momenta. Thus the traces need to be calculated for only one of the two processes. Give the differential cross section for both processes in the Center of Mass frame. Discuss the limiting cases of low and high energy for both processes.

38. S, Problem 48.5

39. **Feynman rules for a Majorana field.** Recall that a Majorana field has the expansion and anticommutation relations

$$\psi(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2\omega}}\sum_\lambda \left(b_\lambda(\boldsymbol{p})u_\lambda(\boldsymbol{p})e^{ix\cdot p} + b^\dagger_\lambda(\boldsymbol{p})v_\lambda(\boldsymbol{p})e^{-ip\cdot x}\right) \tag{2}$$

$$\{\psi^a(\boldsymbol{x}),\psi^b(\boldsymbol{x}')\} = (i\gamma^2)_{ab}\delta(\boldsymbol{x}-\boldsymbol{x}'), \qquad \psi^\dagger = \psi^T i\gamma^2 \tag{3}$$

a) Show that written as $\langle 0|T[\psi_a(x)\bar{\psi}_b(y)]|0\rangle$ the propagator is the same as the Dirac propagator $S_F(x-y)_{ab}$. Obtain from this result an expression for $\langle 0|T[\psi_a(x)\psi_b(y)]|0\rangle$, which is nonzero for a Majorana field (but not for a Dirac field!).

b) Consider an interaction with a scalar field ϕ due to a term $(g/2)\phi\bar{\psi}\psi$ in the Hamiltonian in time dependent perturbation theory, and show that, if ϕ is an external field, to lowest order in g the Feynman amplitude $\mathcal{M}$ is $-ig\bar{u}(\boldsymbol{p}')u(\boldsymbol{p})\tilde{\phi}(p'-p)$ Note that this would be the amplitude for a Dirac interaction term $g\phi\bar{\psi}\psi$. Explain the factor of 2 difference in the Dirac and Majorana interaction terms.

c) Now assume ϕ is a quantum field, and write down the expression for the (connected) second order correction to the momentum space ϕ propagator $\int d^4x e^{iq\cdot x}\langle 0|T\phi(x)\phi(0)|0\rangle$. Compare the expressions for the Majorana and Dirac field and explain the differences. Do all of the position space integrals, but leave the last momentum integral undone.