Standard Model/Quantun Field Theory
Problem Set 1:

Due: 6 September 2019

Suggested reading: QFT Notes Chs 16-18; Textbook: Secs. 58-59,61.

1. **Gaussian Integrals**

a) Let x_k, $k = 1, \ldots, N$ be a set of real integration variables, and let M_{kl} be a real symmetric $N \times N$ matrix. Calculate the integral

$$\int_{-\infty}^{\infty} \prod_k dx_k e^{-(1/2)x_k M_{kl} x_l + iJ_k x_k} \propto (\det M)^{-1/2} e^{(1/2)(iJ_k)(M^{-1})_{kl}(iJ_l)} \tag{1}$$

where the "source" J_k is a set of N real constants. It is understood that repeated indices are summed.

b) Extend this result to integration over Grassmann numbers a_k and $\bar{a}_k$ with source η_k and $\bar{\eta}_k$:

$$\int \prod_k da_k d\bar{a}_k e^{-\bar{a}_k M_{kl} a_l + i\bar{\eta}_k a_k + i\bar{a}_k \eta_k} \propto (\det M) e^{(i\bar{\eta}_k)(M^{-1})_{kl}(i\eta_l)} \tag{2}$$

Here a_k and $\bar{a}_k$ are independent Grassmann variables. See notes chapter 13 for a very brief intro to Grassmann integration.

2. Find the formula, for arbitrary spacetime dimension D, for the degree of divergence $\mathcal{D}$ for a general diagram in:

a) Scalar electrodynamics

$$\mathcal{L}^0_{\rm QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - (\partial + iQ_0 A)\phi^\dagger \cdot (\partial - iQ_0 A)\phi - \mu^2 \phi^\dagger\phi - \frac{\lambda}{4}(\phi^\dagger\phi)^2 \tag{3}$$

b) Hermitian scalar ϕ^3 theory:

$$\mathcal{L} = -\frac{1}{2}((\partial\phi)^2 + \mu^2\phi^2) - \frac{g}{3!}\phi^3 \tag{4}$$

c) Hermitian scalar ϕ^4 theory:

$$\mathcal{L} = -\frac{1}{2}((\partial\phi)^2 + \mu^2\phi^2) - \frac{g}{4!}\phi^4 \tag{5}$$

In each case, draw all the one loop diagrams which are UV divergent in 4 spacetime dimensions. Do the same for the ϕ^3 case in 6 spacetime dimensions.

3. Calculate the differential cross section for

a) The elastic scattering process $e^- + \mu^- \to e^- + \mu^-$ in lowest order perturbation theory in QED, assuming the initial particles are unpolorized and that the spin of final particles is not observed. You may work in the center of mass system.

b) The inelastic scattering process $e^- + e^+ \to \mu^- + \mu^+$ in lowest order perturbation theory in QED, assuming the initial particles are unpolorized and that the spin of final particles is not observed. You may work in the center of mass system.

4. The π^0 is a pseudoscalar strongly interacting particle. We shall see later that its coupling to the electromagnetic field can be described by a term in the Lagrangian of the form

$$G\phi_\pi \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \tag{6}$$

where ϕ_π is a hermitian pseudoscalar field and F is the electromagnetic field strength tensor. Here we are using QFT as an effective theory with G a parameter that should be related to fundamental parameters of QCD. This effective theory should give a good description at low energies. Calculate, to lowest order in G and in the pion rest frame, the differential and total rates for the decay process $\pi_0 \to 2$ photons.