Standard Model/Quantun Field Theory

Problem Set 4

Due: 18 October 2019

Suggested reading: QFT Notes Ch 22; Textbook: Secs. 69-73

10. In the lecture notes we listed the eight Gell-Mann lambda matrices λ_a in Eq (22.19). Then the generators of $SU(3)$ in the (defining) 3 dimensional representation are given by $T_a = \lambda_a/2$.

a) Confirm that these T_a satisfy $\mathrm{Tr} T_a T_b = \delta_{ab}/2$.

b) Use this explicit representation to evaluate the structure constants f_{abc} for $SU(3)$. Remember that the f_{abc} are antisymmetric under the exchange of any pair of indices.

11. Consider a general Lie group with generators T^a, For a general representtion R assume they are orthonormal $\mathrm{Tr} T_R^a T_R^b = T(R)\delta_{ab}$. The Casimir operator is defined by $C(R) = \sum_a T_R^a T_R^a$ or with summation convention $C(R) = T_R^a T_R^a$

a) Show that $[C(R), T_R^b] = 0$. Thus it has the same value on all states in an irreducible representation. E.g. for the rotation group it is just $\boldsymbol{J}^2$ and has the value $j(j+1)$ in a representation of spin j.

b) Let $D(R)$ be the dimension of the representation R, and denote the Adjoint representation by $R = A$. Prove that $T(R)D(A) - C(R)D(R)$.

c) Remembering that the nonabelian field strength transforms in the adjoint representation prove the Bianchi identity:

$$D_\mu F_{\rho\sigma} + D_\rho F_{\sigma\mu} + D_\sigma F_{\mu\rho} = 0$$

12. Using dimensional regularization, calculate the one loop self energy diagram of a Dirac fermion in a general representation of the gauge group coupled to a general (nonabelian) gauge field in the ξ gauge, i.e. the gauge field propagator is

$$-i\delta_{ab} \frac{\eta_{\mu\nu} + (\xi - 1)k_\mu k_\nu / k^2}{k^2 - i\epsilon}. \tag{1}$$

Assume the fermion momentum p is off-shell, i.e. $p^2 \neq -m^2$ so the integral will be finite in the infrared. Calculate the residue of the pole at $D = 4$ and comment on the simplification that occurs for $\xi = 0$ (Landau Gauge).