Standard Model/Quantun Field Theory

Problem Set 5

Due: 1 November 2019

Suggested reading: QFT Notes Ch 22-23; Textbook: Secs. 74,87-88.

13. Consider a pure gauge theory with no extra “matter fields” for example QCD with no quarks. Call the gauge bosons gluons. Draw all the tree diagrams contributing to elastic gluon-gluon scattering and evaluate them, in your favorite gauge, in terms of the polarization vectors of the gluons and their color indices. Check gauge invariance by replacing one of the polarization vectors, say e_1 by p_1 and confirming that this gives 0.

14. In diagonalizing the fermion mass matrices in the standard model we needed to know that any complex square matrix M can be brought to diagonal form with nonnegative diagonal entries by a transformstion $UMV^\dagger$ with U and V a pair of unitary matrices. Prove this.

15. S, Problem 87.2. Note that in the lecture notes we defined $v = \langle\phi_1\rangle$, whereas Srednicki calls this same quantity $v/\sqrt{2}$.

16. Consider the standard model lagrangian using ’t Hooft ξ gauge in the presence of the Higgs mechanism as specified in Eq, (22.81) of the Lecture notes.

a) By examining the terms quadratic in the scalar fields, work out the masses for the fluctuation fields $\hat{\phi} = \phi - v$. Assume $v^T = (v, 0)$, with v real, as we did in class. Show that their squares are proortional to ξ such that the masses coincide with the associated gauge boson masses when $\xi = 1$. Remember that the components of $\hat{\phi}$ are complex.

b) By considering the transformation of the gauge fixing function under infinitesimal gauge transformations work out the FP ghost terms and find the masses of the FP ghosts for general ξ. Compare them to the masses of the scalar field fluctuations obtained in part a).

17. **Decay of the W.**

a) Calculate the decay rate for the processes $W^- \to l + \bar{\nu}_l$ for each of the leptons $l = e, \mu, \tau$ and $W^- \to D + \bar{U}$ for the quarks $D = d, s, b$ and $U = u, c$. Be sure to include the CKM quark mixing matrix. Why do we leave t out of the possible final states?

b) We cannot observe the quark final states, since they are confined inside of hadrons. Instead the produced quarks will be 100% converted into hadrons. Assuming that the total rate for decay into all possible hadronic states is given by the total rate calculation for all possible quark anti-quark pairs, estimate the ratio $\Gamma_{W\to hadrons}/\Gamma_{W\to leptons}$ and compare to the data. You may neglect the allowed quark and lepton masses in comparison with M_W.