Standard Model/Quantun Field Theory

Problem Set 7

Due: 4 December 2019

Suggested reading: QFT Notes Ch 25; Textbook: Sec. 83

22. **The Higgs particle**. As we have seen in class, the way particles are given mass in the standard model is through coupling to the complex scalar Higgs field doublet ϕ which has a vacuum expectation value which we describe by writing

$$\phi = \begin{pmatrix} v + (h + ia)/\sqrt{2} \\ \phi_2 \end{pmatrix} \tag{1}$$

where we take the vacuum value v to be a real number, and h and a are real fields. The complex field ϕ_2 and the real field a provide the zero helicity fields for the massive vector bosons W and Z. The remaining real field h describes a new massive spin zero particle, the Higgs particle. Its mass M_h and its quartic self coupling λ are independent parameters in the standard model.

a) Identify the cubic terms in the standard model Lagrangian that contain one factor of h and two other fields: two vector bosons, quark-antiquark, and lepton-antilepton. Hint: since h enters the Lagrangian with v in the combination $v+h/\sqrt{2}$, you can find these terms by substituting $v \to v + h/\sqrt{2}$ in the mass (quadratic) terms of the Lagrangian and expanding to first order in h. Use the fact that the particle masses are all proportional to v.

b) Extract the Feynman rules for the cubic vertices corresponding to each of these terms.

c) Calculate the decay rate into each of these two body final states, (assuming M_h is large enough for the decay to proceed in each case). Now that the Higgs particle mass is known to be $M_h \approx 125$GeV, the decay into a pair of vector bosons is purely an academic exercise!

23. **Calculating the effective potential.** Consider the field theory of a single real scalar field ϕ with Lagrangian density

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4!}(\phi^2 - a^2)^2, \tag{2}$$

which is invariant under $\phi \to -\phi$. Define the action $S(\phi) \equiv \int d^4x\mathcal{L}$. Recall that the effective action $\Gamma(\varphi) \equiv W(J) - \int d^4x\varphi J$, where $e^{iW(J)}$ is the functional integral over ϕ of $\exp\{iS(\phi) + i\int d^4x\phi J\}$, and $\varphi(x) \equiv \langle\phi\rangle_J = \delta W/\delta J$. For the effective potential it will be enough to introduce a constant source J, so $W(J) = VTw(J)$, and $\varphi = \partial w/\partial J$, and $\Gamma(\varphi) = -VTV_{eff}(\varphi)$.

a) In class we have observed that $W(J)$ is the sum of all connected vacuum graphs (0-point functions) for the action $S(\phi) + \int d^4x\phi J$. By changing variables in the path integral for $e^{i\Gamma(\varphi)}$, show that $\Gamma(\varphi)$ is the sum of all connected vacuum graphs for the action $S(\phi+\varphi) + \int d^4x\phi J$, where J *is chosen so that* $\langle\phi\rangle_J = 0$.

b) In terms of Feynman graphs $\langle\phi(x)\rangle$ is the sum of all connected one-point diagrams ("*tadpoles*") which have a propagator emerging from the point x with the other end hooked to an arbitrary structure with no further external legs. The condition $\langle\phi\rangle_J = 0$ means that the sum of all such tadpole diagrams is zero. But it also means that all diagrams with one or more tadpole subdiagrams will sum to zero. Show that a connected vacuum diagram that is one particle reducible (i.e. can be disconnected by cutting a single line) always has at least one tadpole subdiagram, implying that all one particle reducible diagram contributions to Γ cancel out in the complete sum, so Γ can be calculated as the sum of all connected 1PIR vacuum diagrams.

c) Assuming constant J, φ obtain the Feynman rules for the action $S(\phi+\varphi)+\int d^4x\phi J$. List the propagator, as well as the values of the 1, 3, and 4 point vertices. Note that some of these couplings and the mass occurring in the propagator depend explicitly on φ. Also note that J explicitly appears only in the 1-point vertex. Since this vertex only appears in one particle reducible diagrams the calculation of Γ will have no explicit dependence on J. This means that the complicated relation between φ and J implied by $\langle\phi\rangle_J = 0$ is not needed!

d) The zero loop value for $\Gamma(\varphi)$ is clearly just $S(\phi) = -VT(\lambda/4!)(\varphi^2 - a^2)^2$. The one loop vacuum diagram is represented by a diagram with a propagator closing on itself with no vertices, and stands for the result of doing the Gaussian integral over ϕ using the quadratic terms in the action: $\det^{-1/2}(m(\varphi)^2 - \partial^2)$. For constant φ its contribution to $i\Gamma$ is the log, $-(VT/2)\int(d^4p/(2\pi)^4)\ln(m(\varphi)^2+p^2)$. This diagram is usually ignored when calculating in a source free theory-its just the zero-point energy of the vacuum and has no direct significance. But for the calculation of the effective potential the mass in the propagator depends on φ and that dependence *is* significant. Calculate the effective potential $V_{eff}(\phi)$ through one loop. Do the one-loop integral in Euclidean space (after Wick rotation) with the simple cutoff $p^2 < \Lambda^2$. Show that the cutoff dependence can be absorbed in renormalized parameters. (We know that there must also be field renomalization $\varphi_r = \varphi/\sqrt{Z}$, but with constant φ you won't be able to separate the part of the divergence to be absorbed in Z).

24. As mentioned in class chiral $SU(2)\times SU(2)$ is isomorphic to $O(4)$. In this exercise we explore the relationship further. Let J_L^i, J_R^i be the commuting generators of the two $SU(2)$'s, rotating the L and R components of the quark fields respectively. They each satisfy the usual angular momentum commutation relations.

a) Show that $J^i = J_L^i + J_R^i$ generates the $O(3)$ rotations of ordinary isospin. That is, $\bar{q}q$, $\bar{q}i\gamma_5 q$ are isoscalars and $\bar{q}\boldsymbol{\tau}q$, $\bar{q}i\gamma_5\boldsymbol{\tau}q$ are isovectors under these transformations.

b) Show that $K^i = J_L^i - J_R^i$ acts as a "boost" that transforms $(\bar{q}q, \bar{q}i\gamma_5\boldsymbol{\tau}q)$ and $(\bar{q}i\gamma_5 q, \bar{q}\boldsymbol{\tau}q)$ as 4-vectors. The terminology here is to suggest an analogy with a Lorentz transformation, but of course $O(4)$ is a compact group. The condensate for isospin conserving chiral symmetry breaking is then $\langle\bar{q}q\rangle = v \neq 0$.

c) Work out the commutators of the $O(4)$ generators J^i, K^i among themselves.