Standard Model/Quantun Field Theory III

Problem Set 2

Due: Wenesday, 5 February 2020

Suggested reading: QFT Notes, Ch 25-26; Sr, Secs 75-77,83; P, Ch 19; Sc, Ch 30. Here Sr=Srednicki, P=Peskin&Schroeder, and Sc=Schwartz. These sources cover the same material, but from different poits of view. My notes are self-contained.

4. An important part of the derivation of low energy pion nucleon scattering from spontaneously broken chiral symmetry was establishing that the C terms in the effective action (25.89) in the lecturenotes gave a contribution suppressed by a factor of m_π/m_N compated to the other terms. This was sketched in class Eqs (25.99)-(25.101). Redo this calculation by first confirming that the values for the diagrams quoted in these equations follow from the effective action and then present the calculation supplying all missing steps. Be especially careful with the signs and coefficients needed for the cancellation of the order m_π terms linear in C.

5. Use isospin symmetry to relate pion proton scattering total cross sections to pion neutron total cross sections. Show in particular that, in the limit of exact isospin symmetry, $\sigma_{\pi^+p} - \sigma_{\pi^-p} = \sigma_{\pi^-n} - \sigma_{\pi^+n}$.

6. Anomalies and Instantons

a) For nonabelian auge theory, calculate $\partial_\mu K^\mu$ where

$$K^\mu = \frac{g_3^2 n_F}{4\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left[A_\nu\partial_\rho A_\sigma - \frac{2ig_3}{3}A_\nu A_\rho A_\sigma\right] \tag{1}$$

where $A_\mu = \sum_a t_a A^a_\mu$ is the matrix represntaion of the gauge field, and show that $\partial_\mu K^\mu$ matches the chiral anomaly.

b) Compute the integral

$$\int d^3\theta\epsilon^{ijk}\mathrm{Tr}\left[\Omega^\dagger\partial_i\Omega\Omega^\dagger\partial_j\Omega\Omega^\dagger\partial_k\Omega\right] = \mp 12\pi^2 \tag{2}$$

where $\Omega(\boldsymbol{\theta}) = \pm\sqrt{1-\boldsymbol{\theta}^2}I + i\boldsymbol{\theta}\cdot\boldsymbol{\tau}$ maps S_3 to $SU(2)$.

c) Calculate the field strengths for the one instanton potentials in an $SU(2)$ gauge theory:

$$A_\mu = \frac{i}{g}\begin{cases}\dfrac{-i\boldsymbol{x}\cdot\boldsymbol{\tau}}{r^2+R^2} & \mu = 4\\[2ex] \dfrac{ix^4\tau^k + i\boldsymbol{\tau}\times\boldsymbol{x})^k}{r^2+R^2} & \mu = k\end{cases} \tag{3}$$

and prove that A_μ solve the classical (Euclidean) field equations by showing that the field strengths are self- or anti- dual.

7. Consider the generalization of the winding number formula to general dimension d.

a) Show that

$$\partial_{\mu_1}\epsilon^{\mu_1\cdot\mu_d}\mathrm{Tr}U^\dagger\partial_{\mu_2}U\cdots U^\dagger\partial_{\mu_d}U = \begin{cases} 0 & d \quad \text{odd} \\ -\epsilon^{\mu_1\cdot\mu_d}\mathrm{Tr}U^\dagger\partial_{\mu_1}U\cdots U^\dagger\partial_{\mu_d}U & d \quad \text{even} \end{cases} \tag{4}$$

b) When d is odd, the quantity

$$\omega \quad = \quad \epsilon^{\mu_1\cdot\mu_d}\mathrm{Tr}U^\dagger\partial_{\mu_1}U\cdots U^\dagger\partial_{\mu_d}U \tag{5}$$

is not a total derivative. Show, however, that its variation under $U \to U + \delta U$ to first order in δU is a total derivative. If you wish you may specialize to the special case $d = 5$ which figures in our discussion of the WZW term.