Standard Model/Quantun Field Theory III

Problem Set 5

Due: Wednesday, 25 March 2020

Suggested reading: QFT Notes, Ch 29; Sr, Secs 88-90; P, Ch 21; Sc, Ch 31. Here Sr=Srednicki, P=Peskin&Schroeder, and Sc=Schwartz.

13. Let I_1, I_2, I_3 be the generators for isospin in the vector representation. Prove the identity

$$e^{i\theta I_2} = 1 + iI_2 sin\theta + I_2^2(\cos\theta - 1) \tag{1}$$

by checking it in a basis where I_2 is diagonal. Use it to show that G-parity $G = Ce^{i\pi I_2}$ reversses the sign of all three components of the pion field.

14. There is more to isospin invariance than mass degeneracies. The Δ is a prominent resonance in both $\pi^+ p$ and $\pi^- p$ scattering, that has been determined to have isospin $I = 3/2$. Since the nucleon has $I = 1/2$ and the pion has $I = 1$, the pion nucleon system can have $I = 1/2, 3/2$.

a) Following the familiar angular momentum addition rules express the states $|\pi^+ p\rangle, |\pi^- p\rangle, |\pi^0 n\rangle$ in terms of total isospin states $|I, I_3\rangle$, for $I = 3/2, 1/2$.

b) Let $A_{3/2}$ and $A_{1/2}$ be the pion-nucleon scattering amplitudes in isospin 3/2, 1/2 respectively. Express the amplitudes for the processes $\pi^+ p \to \pi^+ p$, $\pi^- p \to \pi^- p$, $\pi^- p \to \pi_0 n$ in terms of the A's.

c) Use the results of part b) to predict the ratio of total cross sections for $\pi^+ p$ and $\pi^- p$ scattering (in an energy region where we can neglect multiple pion production)

$$\frac{\sigma_{\pi^+ p}}{\sigma_{\pi^- p}} = \frac{3|A_{3/2}|^2}{|A_{3/2}|^2 + 2|A_{1/2}|^2}. \tag{2}$$

The $I = 3/2$ assignment to the Δ means that in the resonance energy region we should have $|A_{1/2}| \ll |A_{3/2}|$, which predicts a ratio of 3. Compare this prediction to the data.

15. π^-, K^- **beta decay** (Note that this problem is moved to Set 6 and need not be handed in with Set 5.)

a) Exploiting the CVC hypothesis for pion beta decay $\pi^- \to \pi^0 + e^- + \bar{\nu}_e$ complete the calculation of the total rate. By comparing to experiment get an estimate of $|V_{ud}|$. For simplicity you may set $m_e = 0$.

b) Kaon beta decay $K^- \to \pi^0 + e^- + \bar{\nu}_e$ involves similar kinematics, but of course involves V_{us} and the estimate of the matrix element based on CVC requires $SU(3)$ symmetry arguments that are less reliable than the isospin symmetry used for pion decay. Nonetheless calculate the rate based on the assumption of exact $SU(3)$ for the matrix element, and get an estimate of $|V_{us}|$ from the data.