Standard Model/Quantun Field Theory III

Problem Set 6

Due: Wednesday, 8 April 2020

Suggested reading: QFT Notes, Ch 28.6-28.8; Sr, Secs 93-94; P, Ch 21; Sc, Ch 29.5. Here Sr=Srednicki, P=Peskin&Schroeder, and Sc=Schwartz.

15. This exercise is to derive the phase space costraints (29.23) in the lecture notes, which follow from the Nambu proposal Action = $T_0\times$ (Area).

a) Parametrize the worldsheet with σ, τ, and define $\dot{x}^\mu = \partial x^\mu/\partial\tau$ and $x'^\mu = \partial x^\mu/\partial\sigma$. Argue that the "area" of the worldsheet can be taken as

$$A = \int d\tau d\sigma \left\{(\dot{x}\cdot x')^2 - \dot{x}^2 x'^2\right\}^{1/2} \tag{1}$$

You may find the formula for the directed area given in Eq(21.33) a useful starting point.

b) Nambu's action is then $S = -T_0 A \equiv \int d\tau d\sigma \mathcal{L}(\dot{x}, x')$. Work out $\mathcal{P}_\mu = \partial\mathcal{L}/\partial\dot{x}^\mu$ and then derive the desired phase space constraints.

16. π^-, K^- **beta decay**

a) Exploiting the CVC hypothesis for pion beta decay $\pi^- \to \pi^0 + e^- + \bar{\nu}_e$ complete the calculation of the total rate. By comparing to experiment get an estimate of $|V_{ud}|$. For simplicity you may set $m_e = 0$.

b) Kaon beta decay $K^- \to \pi^0 + e^- + \bar{\nu}_e$ involves similar kinematics, but of course involves V_{us} and the estimate of the matrix element based on CVC requires $SU(3)$ symmetry arguments that are less reliable than the isospin symmetry used for pion decay. Nonetheless calculate the rate based on the assumption of exact $SU(3)$ for the matrix element, and get an estimate of $|V_{us}|$ from the data.

17. τ **Decay to** $\pi\nu_\tau$ **or** $K\nu_\tau$. The decays of the τ lepton into a pseudoscalar meson and a ν_τ are completely given, to lowest order in G_F, in terms of the meson decay constants as measured in the meson decay into electron or muon plus neutrino. Calculate the rates for $\tau \to \pi^- + \nu_\tau$ and $\tau \to K^- + \nu_\tau$. Compare your results with experiment.