Standard Model/Quantun Field Theory III

Problem Set 7

Due: Wednesday, 22 April 2020

Suggested reading: QFT Notes, Ch 29.5-29.6 and 30; Sr, Secs 93-94; P, Ch 21; Sc, Ch 29.5. Here Sr=Srednicki, P=Peskin&Schroeder, and Sc=Schwartz.

18. **Nonleptonic baryon decay** Test the $\Delta I = 1/2$ rule for the isoscalar Λ baryon decays into $p\pi^-$ and $n\pi_0$ by expressing their rates in terms of $I = 1/2$ and $I = 3/2$ amplitudes. Compare the ratio of rates calculated with the assumption of pure $\Delta I = 1/2$ transitions to the ratio extracted from the data.

19. Fill in some gaps in our discussion of the box diagram model of neutral kaon mixing (Section 29.5.2 of the lecture notes).

a) Calculate the integral encountered in our discussion of $K, \bar{K}$

$$A(x_i, x_j) = \frac{M_W^2}{\pi^2} \int \frac{d^4k_E}{(k_E^2 + M_W^2)^2} \frac{k_E^2/D}{(k_E^2 + m_i^2)(k_E^2 + m_j^2)} \tag{1}$$

to establish the last line of Eq.(29.83) in the lecture notes. Here $x_i = m_i^2/M_W^2$.

b) Derive the approximate formula (Eq.(29.86))

$$\begin{aligned}\sum_{i,j} \xi_i \xi_j A_{i,j} &= \xi_u^2 x_c + \xi_t^2 \left(\frac{x_t + x_t^2}{(1 - x_t)^2} + \frac{2x_t \ln x_t}{(1 - x_t)^3} \right) \\ &\quad + 2\xi_u \xi_t x_c \left(\frac{x_t}{(1 - x_t)} + \frac{\ln x_t}{(1 - x_t)^2} - \ln x_t \right)\end{aligned} \tag{2}$$

c) Put in the experimental numbers and check Eq.(29.94).

20. Deep inelastic scattering os the process $e+p \to e+X$ where X represents an arbitrary hadronic state. Only the final electron is observed, meaning that the squared amplotude is summed over all possible X. We work to lowest order in QED perturbation theory but nonperturbatively in the strong interactions (QCD). The Feynman amplitude for a fixed X can be expressed as the product of the electron spinor bilinear $\bar{u}'\gamma^\mu u$ and the matrix element of the electromagnetic current between X and the proton, $\langle X|j_\mu(0)|p\rangle$. Of course the familiar factors $1/[(2\pi)^{3/2}\sqrt{2E_i}$ for each external particlw in the matrix element are dropped in forming the Feynman amplitude. The squared Feynman amplitude averaged over proton spins and summed over all X can be written as

$$\frac{e^4}{q^4} \bar{u}'\gamma^\mu u \bar{u}\gamma^\nu u' W_{\mu\nu}(q, p) \tag{3}$$

where p is the proton momentum, $q = k - k'$ is the momentum transferred from the electron to the proton.

a) Show that Lorentz covariance and current conservation determine W in terms of two structure functions of q^2 and $q \cdot p$:

$$\begin{aligned} W^{\mu\nu} &= \left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) W_1(q^2, \nu) \\ &\quad + \left(p^\mu - q^\mu \frac{q \cdot p}{q^2}\right) \left(p^\nu - q^\nu \frac{q \cdot p}{q^2}\right) W_2(q^2, \nu) \end{aligned} \tag{4}$$

where $m_p \nu = -q \cdot p$.

b) Calculate the differential cross section, summed and averaged over electron spins, for deep inelastic electron scattering in the proton rest frame. Express your answer in tems of the structure functions $W_{1,2}$, the initial electron energy E_e, and the electron scattering angle θ.